

DESIGN OF A ROBUST MODEL REFERENCE ADAPTIVE VOLTAGE CONTROLLER FOR AN ELECTRODYNAMIC SHAKER

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Abstract – This paper presents in detail the design of a high performance sinusoidal voltage controller for switching-mode AC power amplifiers intended to drive electrodynamic shakers. To obtain good reference tracking performance and stability over a wide frequency range and guarantee robustness under the effects of parameter variations and harmonic distortions, a model reference adaptive control scheme is employed. A very simplified model of the electrodynamic shaker is considered. The back electromotive force is dealt as unknown disturbance and the inductance of the armature is not modeled. The control strategy is digitally implemented and the design procedure requires only basic information about nominal electrical parameters of the shaker and the power amplifier. Fixed and swept frequency experimental tests performed up to 2 kHz demonstrate the feasibility of this solution. The implementation of the voltage controller is also described.

Keywords - Discrete-Time Control, Pulse-Width Modulated Inverters, Robust Adaptive Control, Shakers.

I. INTRODUCTION

Mechanical vibration testing is widely used to study the effects of vibration and to evaluate physical properties of materials and structures in a variety of applications that range from circuit boards and aircraft to turbines and home appliances. Generally, the purpose is to simulate the dynamic environment where the test article will be stored or used in service and then to find out if it will survive to the rigors of the storage or service conditions [1].

The equipment normally needed to execute vibration tests includes an electrodynamic vibration machine (shaker), a power amplifier and an acceleration controlling and monitoring system (Fig. 1). The test article is attached to the table of the shaker and an accelerometer mounted on this table is used for feedback purposes in a vibration control system. The output command r generated by the acceleration controller is then employed as the power supply reference signal.

The main requirements of amplifiers designed to drive electrodynamic shakers are to present good voltage or current control characteristic and low harmonic distortion over a wide dynamic and frequency range (typically, 10 Hz to 2 kHz). Depending on whether the amplifier output current i_o or voltage v_o is proportional to the reference voltage r , the mode of operation is characterized: current (or transconductance) and voltage mode, respectively. In current mode, the mechanical resonance of the shaker suspension is in general very lightly damped, which makes the vibration control pro-

blem even more complicated and restricts this mode to very specific cases where closed-loop acceleration control is not required (such as in modal testing). In voltage mode, on the other hand, the suspension resonance is strongly damped as long as the impedance of the shaker is low. This important feature was determinant to consolidate the voltage mode in the majority of the vibration tests [8].

In terms of voltage control of switching-mode power amplifiers, most of the developed control techniques address the requirements of good reference tracking and low harmonic distortion for low frequency applications such as motor drives and uninterruptible power supplies [2]-[5]. In case of electrodynamic shakers, however, good performance is required over a very higher dynamic and frequency range, which imposes some unusual demands such as the ability of dealing during the same test with the back electromotive force produced in the shaker armature coil (which increases its impedance according to the mass of the test load) and the effects of the high frequency resonance of the amplifier output filter. In addition, due to the wide frequency power excitation, the armature electrical parameters are not constant, which significantly change the nature of the shaker impedance according to the controlled frequency.

The literature reports only a few solutions about the control of power amplifiers intended to drive electrodynamic shakers. In [6] and [7], an analog sinusoidal current control scheme has been presented. This solution is based on a high switching frequency and relatively low output power PWM inverter (50 kHz, 400 VA). Despite of the good control characteristic demonstrated experimentally until 2 kHz, this proposal has a very restricted application range as a consequence of the current mode operation.

A more general solution consisting of a robust model reference adaptive voltage controller has been presented in [9]. This technique comprises a reduced-order model of the electrodynamic shaker, where the back electromotive force is dealt as unknown disturbance and the inductance of the armature is not modeled. The control strategy is digitally implemented and experimental tests performed in a 3 kVA, 12 kHz PWM inverter have demonstrated the feasibility of this solution for controlling the shaker sinusoidal voltage, but only at frequencies up to 1 kHz.

Then, to further explore the applicability of [9], this paper presents some modifications that expand to 2 kHz the frequency range of the voltage controller. Differently from [9], the control structure, the parameter adaptation law and the design of the robust model reference adaptive controller are presented in detail. Some implementation features along with fixed and swept frequency experimental tests are also provided. The development of the acceleration control system is not approached in this paper.

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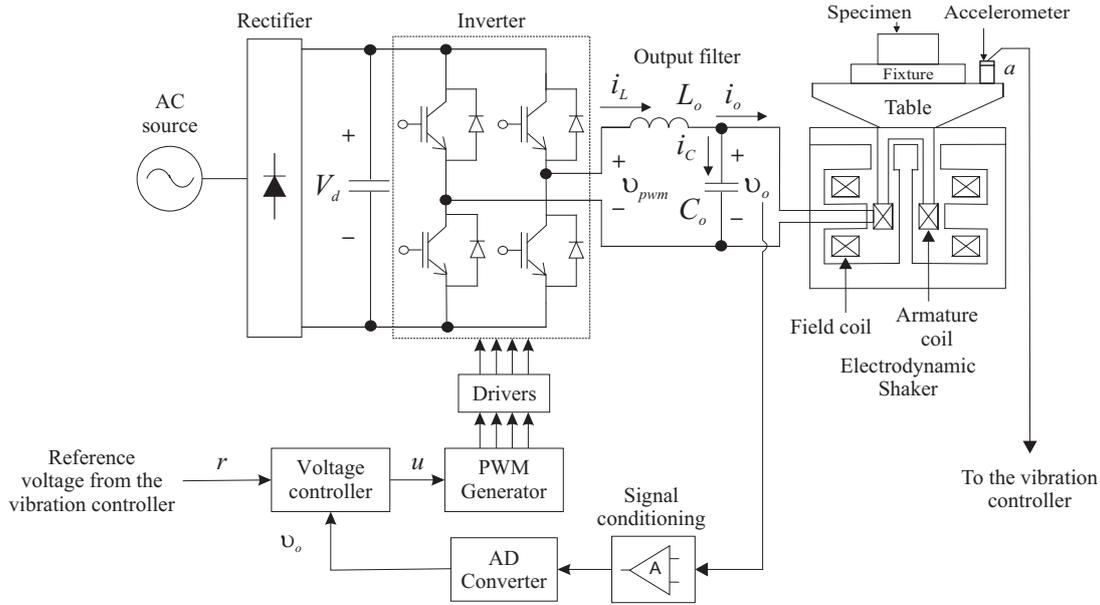


Fig. 1. The switching-mode AC power amplifier for excitation of an electrodynamic shaker.

II. PLANT DESCRIPTION

The schematic diagram of a switching-mode power amplifier designed to excite an electrodynamic shaker is shown in Fig. 1. It consists of a single-phase PWM inverter, a LC filter and a measuring and control platform. The inverter generates a pulse-width modulated voltage v_{pwm} based on the control signal u . The output filter is used to reduce the harmonic content of the shaker input voltage v_o in order to eliminate vibrations outside the main frequency range. The measuring and the control platform acquire the output voltage, compute the control law and generate the PWM signals.

The dynamic behavior of electrodynamic shakers is, in general, very complex due to resonances of the shaker body and moving mass. Unpredictable mechanical resonances and anti-resonances usually occur when elastic structures are attached to the table. The shaker electrical behavior, on the other hand, is determined not only by the moving coil resistance and inductance, but also by the back electromotive force generated across the coil. The mass of the device under test is variable and the coil resistance and inductance are not constant due to the wide frequency range excitation.

For voltage control purposes, however, the shaker impedance is not meaningfully affected by unknown mechanical resonances, which means that the modeling effort can be simplified, especially assuming that a robust control technique comprising reduced knowledge of the plant is to be developed. In this case, the following assumptions are stated:

- (i) The radial magnetic flux density is constant.
- (ii) The shaker body is rigidly attached to the floor.
- (iii) Displacements between the shaker coil, table, fixture and test load are not modeled.

By doing so, the shaker moving element is dealt as a rigid mass and its dynamic behavior is approximately represented by the single degree-of-freedom system of indicated in Fig. 2(a). The mechanical and electrical equations that govern this system are:

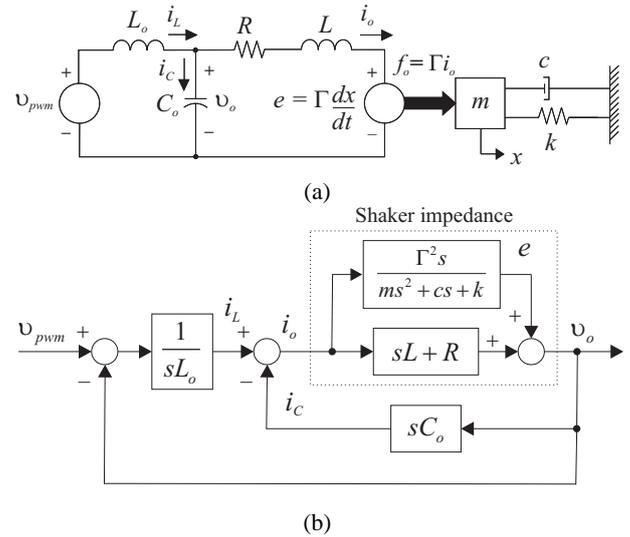


Fig. 2. A very simplified dynamic model: (a) Equivalent electro-mechanical circuit. (b) Transfer function block diagram.

$$v_{pwm} = L_o \frac{di_L}{dt} + v_o \quad (1)$$

$$v_o = L \frac{di_o}{dt} + Ri_o + e \quad (2)$$

$$e = \Gamma \frac{dx}{dt} \quad (3)$$

$$C_o \frac{dv_o}{dt} = i_c = i_L - i_o \quad (4)$$

$$f_o = \Gamma i_o = m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx \quad (5)$$

Where:

Γ - force-generating constant ($\Gamma \triangleq Bl$).

B - magnetic flux density.

l - effective length of the armature conductors.

- e - back electromotive force (back EMF).
- f_o - linear force developed by the armature.
- L_o - inductance of the output filter.
- C_o - capacitance of the output filter.
- v_{pwm} - PWM inverter output voltage.
- v_o - power amplifier output voltage.
- i_o - power amplifier output current.
- i_L - inductor current of the output filter.
- i_C - capacitor current of the output filter.
- L - inductance of the armature coil.
- R - resistance of the armature coil.
- x - displacement of the armature.
- v - vibration velocity of the armature ($v = dx/dt$).
- a - vibration acceleration of the armature ($a = d^2x/dt^2$).
- m - total moving mass.
- c - damping coefficient of the armature suspension.
- k - stiffness of the armature suspension.

Despite most of these equations rely on basic electromechanical principles, special analysis is required for the back EMF. As the armature coil moves within the radial magnetic flux, a voltage is generated across the coil in proportion to the velocity of motion (3), which gives the active nature to the shaker. For a constant amplitude sinusoidal acceleration a , the motional EMF dominance in the equivalent circuit fades as the frequency increases, owing to the continual decrease in velocity ($v = a/(2\pi f)$, where f is the vibration frequency). Then, the armature coil reactance and resistance drops become significant and finally dominant. At very low frequencies (0 to 10 Hz), the inductance drops are negligible and the equivalent circuit approximates to R in series with the EMF generator e . At the suspension mass-spring resonance (typically 20 Hz), motion can be sustained with virtually no current. Beyond this point, the EMF generator has a value inversely proportional to the vibration frequency. Above 1 kHz or so, the equivalent circuit can then be approximated to one comprising R and L in series [11].

The back electromotive force reflects, in summary, the mechanical activity into the electrical circuit so that the shaker impedance changes from a predominantly capacitive load at low frequencies to a highly inductive nature over the high frequency range. Fig. 2(b) shows a linear transfer function block diagram of the corresponding switching-mode AC power amplifier when driving the electrodynamic shaker. The switching frequency is assumed to be much higher than the modulation frequency of the PWM inverter. Note that the moving mass m influences e and, since most sinusoidal vibration tests usually require constant magnitude acceleration with frequency variation at a specified rate in Hz or octave per minute, the amplitude of the back EMF and the shaker electrical parameters R and L are also time variant.

III. ROBUST MODEL REFERENCE ADAPTIVE VOLTAGE CONTROLLER

A. Structure of the Controller

Although electrodynamic shakers present a high degree of linearity between the input voltage v_o and current i_o if used within their specified force and motion limits, the operation over a wide frequency range combined to the meaningful

variations on the shaker characteristics make it difficult to achieve good reference tracking and low harmonic distortion by using only fixed-gain controllers such as proportional-integral-derivative (PID), PI, or P controller. An adaptive control scheme capable to automatically tune its gains and guarantee good performance and stability independently of the uncertainties presented, on the other hand, is recommended for such a case where the plant is strongly subjected to the effects of parameter variations. In this sense, the modeling effort can be further minimized by considering that:

(iv) The back EMF e is an unknown disturbance.

(v) The inductance L of the armature coil is not modeled.

Under these assumptions, the electrodynamic shaker is dealt as the resistive load R shown in Fig. 3, i.e., the effect of e into the output voltage v_o is supposed to be mitigated by the robust adaptive controller and the effect of v_{pwm} can be approximately represented as:

$$G_v(s) = \frac{v_o(s)}{v_{pwm}(s)} = G_{vo}(s)[1 + \mu\Delta_m(s)] \quad (6)$$

where $G_v(s)$ is obtained by letting e equal to zero in the block diagram of Fig. 2(b), i.e.:

$$G_v(s) = \frac{sL + R}{s^3L_oC_oL + s^2L_oC_oR + s(L_o + L) + R} \quad (7)$$

and the modeled part of the plant $G_{vo}(s)$ results by making L equal to 0 in (7):

$$G_{vo}(s) = \frac{R}{s^2L_oC_oR + sL_o + R} \quad (8)$$

Then, $\mu\Delta_m(s)$ is a structured uncertainty on the model of the plant obtained by deliberately neglecting the inductance of the armature, i.e., $\mu = L$ is the neglected parameter and:

$$\begin{aligned} \mu\Delta_m(s) &= \frac{G_v(s) - G_{vo}(s)}{G_{vo}(s)} \\ &= \frac{s^2L_o\mu}{R[s^3L_oC_o\mu + s^2L_oC_oR + s(L_o + \mu) + R]} \end{aligned} \quad (9)$$

is the resulting multiplicative modeling error.

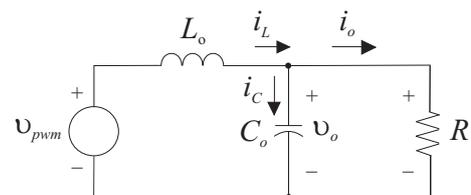


Fig. 3. The nominal load of the AC power amplifier.

The inverter power switches are turned on and off at each sampling interval T so that v_{pwm} is a voltage pulse of magnitude $-V_d$ or V_d and width ΔT . This analysis, however, assumes that the inverter modulator signal u of Fig. 1 (control signal) is the average value of the voltage pulse in a sampling period. Thus, a discrete-time transfer function of (8) can be easily obtained by using a zero-order-hold with an appropriate sampling time [12]:

$$G_{vo}(z) = \frac{v_o(z)}{u(z)} = k_p \frac{Z_o(z)}{R_o(z)} = k_p \frac{z + b_1}{z^2 + a_1z + a_2} \quad (10)$$

where $Z_o(z)$ and $R_o(z)$ are monic polynomials of degree m_p and n_p , respectively.

A model reference adaptive controller applicable to reduced-order plants and robust with respect to multiplicative stable modeling errors and bounded disturbances has been developed in [13] and [14]. This scheme, however, is only applicable to plants satisfying the following assumptions:

A1: $Z_o(z)$ is a monic Hurwitz polynomial of degree $m_p \leq n_p - 1$;

A2: $R_o(z)$ is a monic polynomial of degree n_p ;

A3: the sign of k_p and the values of m_p and n_p are known;

A4: $\Delta_m(z)$ is a stable transfer function;

A5: An upper bound $1 > p_o > 0$ on the stability margin $p > 0$ for which the poles of $\Delta_m(z/p)$ are stable is known.

For the modeled part of the plant, assumptions A1 to A3 are easily verified according to (10). Assumptions A4 and A5, on the other hand, are satisfied based on a discrete-time representation of $\mu\Delta_m(s)$.

The adaptive control objective consists, basically, in determining an appropriate control parameter vector θ so that the resulting closed-loop plant is stable and the plant output v_o tracks the reference model output v_m as closely as possible, i.e., the desired closed-loop performance is clearly expressed in terms of the reference model choice:

$$W_m(z) = \frac{v_m(z)}{r(z)} = k_m \frac{Z_m(z)}{R_m(z)} \quad (11)$$

where $Z_m(z)$ and $R_m(z)$ are arbitrary monic Hurwitz polynomials of degrees m_p and n_p , respectively, and r is an uniformly bounded reference.

To achieve the adaptive control objective, the input u and the output v_o are used to generate the following auxiliary vectors:

$$\omega_1(k+1) = \mathbf{F}_d \omega_1(k) + \mathbf{q}_d u(k) \quad (12)$$

$$\omega_2(k+1) = \mathbf{F}_d \omega_2(k) + \mathbf{q}_d v_o(k)$$

where \mathbf{F}_d is a $(n_p - 1) \times (n_p - 1)$ stable matrix and $(\mathbf{F}_d, \mathbf{q}_d)$ is a controllable pair. Then, the input control law results as:

$$u(k) = \theta^T(k) \omega(k) + c_o r(k) \quad (13)$$

where $\theta^T(k) = [\theta_1^T(k) \quad \theta_2^T(k) \quad \theta_3(k)]$ is the $(2n_p - 1)$ control parameter vector, $\omega^T(k) = [\omega_1^T(k) \quad \omega_2^T(k) \quad v_o(k)]$ and c_o is a scalar feedforward parameter.

The augmented error ε_1 used for parameter adaptation can be stated as in [13]:

$$\varepsilon_1(k) = v_o(k) - v_m(k) + \theta^T(k) \zeta(k) - v(k) \quad (14)$$

where $\zeta = W_m(z) \mathbf{I} \omega$ and $v = W_m(z) \theta^T \omega$.

Fig. 4 shows a block diagram of the corresponding robust model reference adaptive voltage controller. Note that the control parameters are tuned based on the algorithm described next. Also, according to [15], the pair $(\mathbf{F}_d, \mathbf{q}_d)$ corresponds to a discrete-time approximation of (\mathbf{F}, \mathbf{q}) , which in turn is a state space realization of $\alpha(s)/\Lambda(s)$, i.e.:

$$(s\mathbf{I} - \mathbf{F})^{-1} \mathbf{q} = \frac{\alpha(s)}{\Lambda(s)} = \frac{\omega_1}{u} = \frac{\omega_2}{v_o} \quad (15)$$

where $\Lambda(s)$ is an arbitrary monic Hurwitz polynomial of degree n_p whose roots are commonly designed based on the desired closed-loop bandwidth.

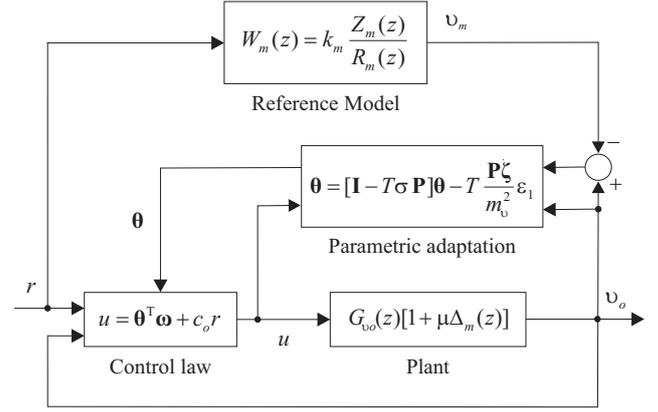


Fig. 4. The robust model reference adaptive controller.

B. Parameter Adaptation Algorithm

A modified least-squares algorithm subjected to assumptions A1 to A5 has been presented in [16]. The proposed solution exhibits fast parameter convergence and robustness even under the effects of time delays usually present in digital implemented control systems. In this case, a recursive form can be obtained by using the forward difference approximation:

$$\theta(k+1) = [\mathbf{I} - T\sigma\mathbf{P}(k)]\theta(k) - T \frac{\mathbf{P}(k)\zeta(k)}{m_v^2(k)} \varepsilon_1(k) \quad (16)$$

$$\mathbf{P}(k+1) = -T \left[\frac{\mathbf{P}(k)\zeta(k)\zeta^T(k)\mathbf{P}(k)}{m_v^2(k)} + \frac{\mathbf{P}^2(k)}{R_v^2} \bar{\mu}^2 \right] + (1 + T\lambda\bar{\mu}^2)\mathbf{P}(k) \quad (17)$$

where $\mathbf{P}(k) = \mathbf{P}^T(k)$ is such that:

$$0 < \mathbf{P}(0) \leq \lambda R_v^2 \mathbf{I}, \quad \mu^2 \leq k_\mu \bar{\mu}^2 \quad (18)$$

$$m_v(k+1) = (1 - T\delta_0)m_v(k) + T\delta_1 (|u(k)| + |v_o(k)| + 1) \quad (19)$$

$$m_v(0) > \frac{\delta_1}{\delta_0}$$

and $\lambda, \bar{\mu}, R_v, \delta_0, \delta_1$ are positive constants, $\delta_1 \geq 1$ and $\delta_0 < 1$ satisfies:

$$\delta_0 \delta_2 > \max[p_0, q_0] \quad (20)$$

where δ_2 is a positive constant and $1 > q_0 > 0$ is such that the poles of $W_m(z/q_0)$ and $[(z/q_0)\mathbf{I} - \mathbf{F}_d]^{-1}$ are stable, p_0 is defined in A5 and the σ -modification function is given by:

$$\sigma = \begin{cases} 0 & \text{if } \|\theta(k)\| < M_0 \\ \sigma_0 \left(\frac{\|\theta(k)\|}{M_0} - 1 \right) & \text{if } M_0 \leq \|\theta(k)\| \leq 2M_0 \\ \sigma_0 & \text{if } \|\theta(k)\| > 2M_0 \end{cases} \quad (21)$$

where M_0 is an upper bound for the norm of the desired control parameter vector θ^* and $\sigma_0 > 2\bar{\mu}^2/R_v^2$.

C. Controller Design

The design of the adaptive controller consists, mainly, in determining the desired closed-loop performance in terms of the reference model choice. Usually, there is no predefined

rule for designing $W_m(z)$ but a common approach relates the reference model to the dynamic of the open loop system.

As an example, the electrodynamic shaker model St 5000/300 (manufactured by TIRA) and the power amplifier (Semikron, model SK45GB063) are considered. Fig. 5 shows the experimental variations of the shaker resistance R and inductance L produced as consequence of the skin effect. The design of the voltage controller, however, considers only the nominal parameters indicated in Table I (symbols with over line bar indicate nominal value).

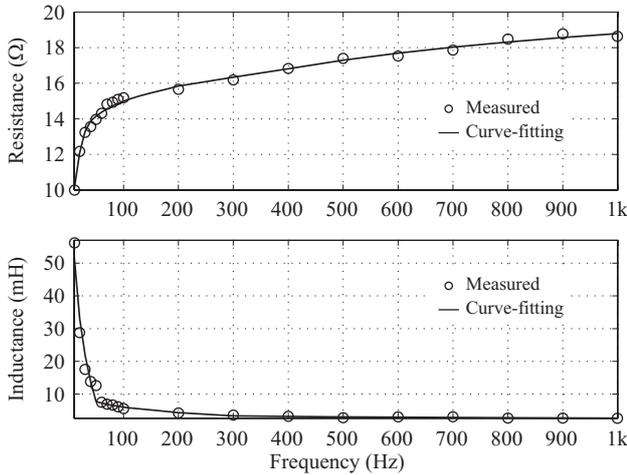


Fig. 5. The armature resistance and inductance.

TABLE I
Nominal electrical parameters

\bar{L}_o (μH)	\bar{C}_o (μF)	\bar{R} (Ω)
250	10	12

Based on these parameters, (8) becomes:

$$G_{vo}(s) = \frac{4 \times 10^8}{s^2 + 8.33 \times 10^3 s + 4 \times 10^8} \quad (22)$$

Then, selecting $W_m(s)$ with smaller rise and settling time than $G_{vo}(s)$ usually results in good closed-loop performance and avoids overloading the actuator during transient periods:

$$W_m(s) = \frac{9.87 \times 10^8}{s^2 + 3.96 \times 10^4 s + 9.87 \times 10^8} \quad (23)$$

A comparison between the step responses of $G_{vo}(s)$ and $W_m(s)$ in Fig. 6 indicates that the performance of the closed-loop system is now mainly dependent on how the nominal model of the plant represents the actual electrodynamic shaker and, more importantly, on how the parameter adaptation algorithm can deal with differences.

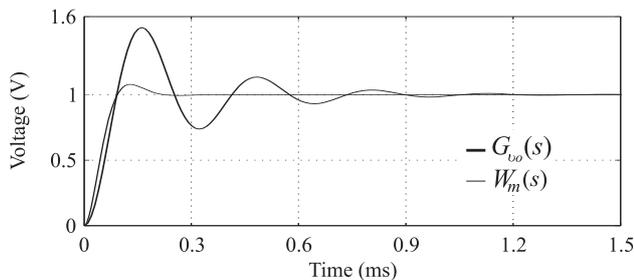


Fig. 6. Step responses of $G_{vo}(s)$ and $W_m(s)$.

The digital implementation of the adaptive controller is straightly linked to the sampling frequency choice. A good rule for setting the sampling time T consists in determining the bandwidth of the desired closed-loop system and then to make $1/T$ about 6 to 25 times greater than the corresponding bandwidth [12]. Obviously, the sampling rate influences properties like the command signal tracking, the rejection of disturbances and measurement noise, the sensitivity to unmodeled dynamics and the computer power required.

In the current case, the system is designed to operate according to the low frequency limit of the electrodynamic shaker (20 Hz) and the upper limit of the power amplifier (2 kHz). Then, for ease of implementation, the sampling frequency ($1/T$) is chosen equal to the inverter switching rate (24 kHz) so that a balance between the closed-loop bandwidth criteria and the switching frequency is achieved.

The discrete-time equivalent representation of (22) and (23) is obtained by applying z -transformation to the zero-order-hold/transfer function product, i.e.:

$$G_{vo}(z) = 0.293 \frac{z + 0.888}{z^2 - 1.153z + 0.707} \quad (24)$$

$$W_m(z) = 0.467 \frac{z + 0.565}{z^2 - 0.4614z + 0.1922} \quad (25)$$

For a second order plant, the filter $\alpha(s)/\Lambda(s)$ of u and v_o in (15) is defined as in [15]:

$$\frac{\alpha(s)}{\Lambda(s)} = \frac{q}{s - F} \quad (26)$$

and the corresponding digital implementation (F_d, q_d) is determined by applying a zero-order-hold to $(sI - F)^{-1}q$:

$$F_d = e^{FT} \quad (27)$$

$$q_d = (e^{FT} - 1) \frac{q}{F}$$

To present a unitary gain at low frequencies and to avoid high frequency noise amplification in the control law, $\alpha(s)/\Lambda(s)$ is designed with q set equal to $-F$ and the cutoff frequency $-F$ is defined as 2.000 rad/s. Then, F_d and q_d result from (27) as 0.92 and 0.08, respectively.

The modeled part $G_{vo}(s)$ of the plant has a magnitude increase at high frequency operation due to the resonance that occurs between the output filter inductor and capacitor. In practice, this amplification can be dealt as an increase of k_p and necessity of reducing the control parameter c_o as long as k_m remains constant (according to the model reference control theory [15], $c_o = k_m/k_p$). Then, differently from [9]:

$$c_o(f) = \begin{cases} 2.2 - 1.7^{f/500} & \text{if } f \leq 500 \\ 0.5 & \text{if } f > 500 \end{cases} \quad (28)$$

where f is the frequency of the reference r .

For the parameter adaptation algorithm, the upper limits on the stability margin p_o and q_o need to be determined firstly. In case of q_o , considering the characteristic equations of $W_m(z)$ and $\alpha(z)/\Lambda(z)$ and substituting z by $q_o z$, i.e.:

$$q_o^2 z^2 - 0.4614 q_o z + 0.1922 = 0 \quad (29)$$

$$q_o z - F_d = 0 \quad (30)$$

the upper limit on the stability margin corresponds to the smallest value of q_o between 0 and 1 so that the roots z_1 and

z_2 of (29) and the root z_3 of (30) satisfy $|z_i| < 1$ ($i = 1, 2$ and 3). Then, by making q_0 to vary from 1 to 0:

$$q_0 = 0.92 \quad (31)$$

and both solutions of (29) and (30) lie within the unit circle of the z -plane.

The upper limit on the stability margin p_0 of $\mu\Delta_m(z)$, on the other hand, is related to the magnitude of the neglected parameter. Although the inductance of the armature is frequency dependent, the stability analysis is more representative by considering only the maximum value of L , which is measured at the lower frequency limit, i.e., $\bar{L} = 55$ mH for model St 5000/300 in Fig. 5. Thus, from (9) and Table I:

$$\mu\Delta_m(s) = \frac{8.33 \times 10^3 s^2}{s^3 + 218.2s^2 + 4.02 \times 10^8 s + 8.73 \times 10^{10}} \quad (32)$$

or, equivalently:

$$\mu\Delta_m(z) = 0.3068 \frac{z^2 - 2z + 1}{z^3 - 2.333z^2 + 2.33z - 0.991} \quad (33)$$

and the upper bound on the stability margin p_0 is obtained similarly to q_0 , but from the characteristic equation of $\mu\Delta_m(z)$:

$$p_0 = 0.99 \quad (34)$$

Based on (20), δ_0 is defined equal to 0.991 and, since $\bar{\mu}$ is an upper bound for L , $\bar{\mu}$ is set equal to 100 mH. Due to the absence of a deterministic design procedure, other positive constants were determined arbitrarily according to the restrictions stated in (16) to (21) and evaluated by computer simulation under different conditions until good convergence rate was obtained. As an example, the initial value of the control parameter vector $\theta(0)$ was firstly set equal to $\mathbf{0}$. Then, after some iterations, it was verified that good convergence is achieved by making $\theta(0)$ equal to $[-1 \ 0.3 \ 0.7]^T$. In case of the desired control parameter vector θ^* , it is assumed that an upper bound M_0 for $\|\theta^*\|$ is known. Nevertheless, it is shown in [13] that choosing M_0 as large as possible but smaller than $1/\bar{\mu}$ always satisfies $M_0 > \|\theta^*\|$ in presence of unmodeled dynamics, provided that $\bar{\mu}$ is small. Table II presents the designed controller parameters.

TABLE II
Designed controller parameters

Symbol	Value
$\theta(0)$	$[-1 \ 0.3 \ 0.7]^T$
$\mathbf{P}(0)$	$100 \times \mathbf{I}_{3 \times 3}$
$\zeta(0)$	$[0 \ 0 \ 0]^T$
F_d	0.92
q_u	0.08
$\omega_1(0)$	0
$\omega_2(0)$	0
λ	10
$\bar{\mu}$	0.1
R_v	10
δ_0	0.991
δ_1	1
M_0	9
σ_0	0.1
T	41.66 μs

IV. IMPLEMENTATION AND EXPERIMENTAL RESULTS

A. The System Configuration and Digital Implementation

The electrodynamic shaker considered in the controller design is a 1 kVA vibration machine capable to operate from 20 Hz to 5 kHz. The maximum sine force peak is approximately 3 kN, the mass of the armature and fixture assembly is equal to 22 kg and the maximum specimen weight is 46 kg. The power amplifier, on the other hand, generates sinusoidal voltages between 10 Hz and 2 kHz with a maximum peak of the output voltage v_o equal to 110 V. Then, based on the vibration machine and the power supply constraints, the frequency range was set equal to 20 Hz to 2 kHz and the maximum voltage and current amplitudes were defined as 110 V and 20 A, respectively.

A multifunction interface board was used to communicate the PC-based control platform and the AC power amplifier. The control routine was developed in Borland C programming language to acquire the external variables and to compute the control law according to the specified voltage amplitude and frequency. The control signal u was converted into a PWM signal and then applied to the AC power amplifier module through the multifunction interface board (Fig. 1). The robust adaptive algorithm was implemented as:

- 1) Calculate $r(k)$ and $v_m(k)$;
- 2) Acquire the external variable $v_o(k)$;
- 3) Compute the control law $u(k)$;
- 4) Generate the PWM signals;
- 5) Calculate $\omega_1(k+1)$, $\omega_2(k+1)$, $\varepsilon_1(k+1)$, σ , $\theta(k+1)$, $\mathbf{P}(k+1)$, $\zeta(k+1)$, $v(k+1)$ and $m_v(k+1)$.

B. Experimental Results

The experimental performance of the robust adaptive voltage controller has been evaluated first with a 24 Ω resistive load. Figs. 7(a) and 7(b) show the corresponding power amplifier output voltage v_o and the reference model v_m at 20 Hz and 2 kHz, respectively. Then, the control scheme has been applied to compensate the tracking error between v_o and v_m when the electrodynamic shaker is inserted (see Fig. 8). Fig. 9 shows results at different amplitudes. Note that due to the limited number of points acquired per cycle in high frequencies, the voltage waveform appears to be distorted in 2 kHz, which is absolutely not verified in the analog domain.

To verify the transient tracking response with frequency varying continuously, a constant amplitude voltage has been employed to drive the electrodynamic shaker. The resonant search test requirements specified in [17] have been taken as a practical example of frequency sweep. According to these specifications, the device under test must be submitted to a sinusoidal logarithmic frequency sweep at the rate of 1 octave per minute, which demands approximately 7 minutes to complete a whole sweep between 20 Hz and 2 kHz. Then, to facilitate the exposition of the results obtained, some particular frequencies were selected and combined in the same figure. Figs. 10(a) and 10(b) show the corresponding measured voltage waveforms from 30 Hz to 100 Hz and from 600 Hz to 2 kHz, respectively.

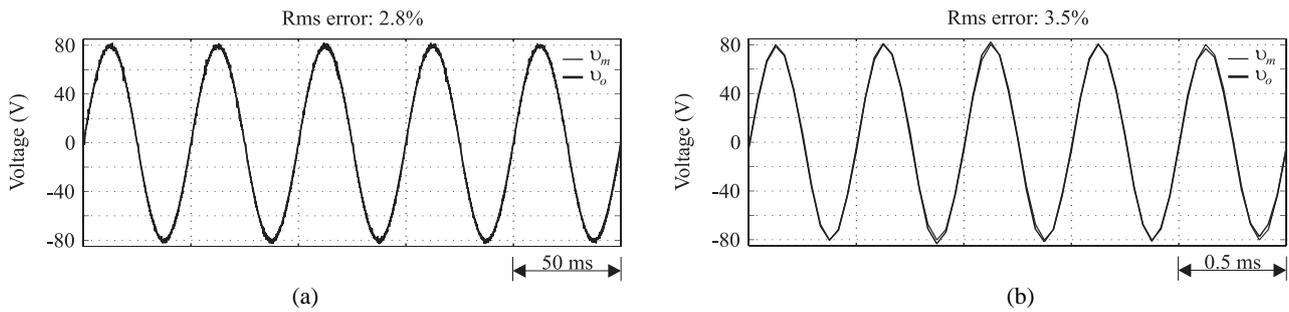


Fig. 7 Reference model output v_m and the measured voltage waveform v_o with a 24Ω resistive load. (a) 20 Hz. (b) 2 kHz.

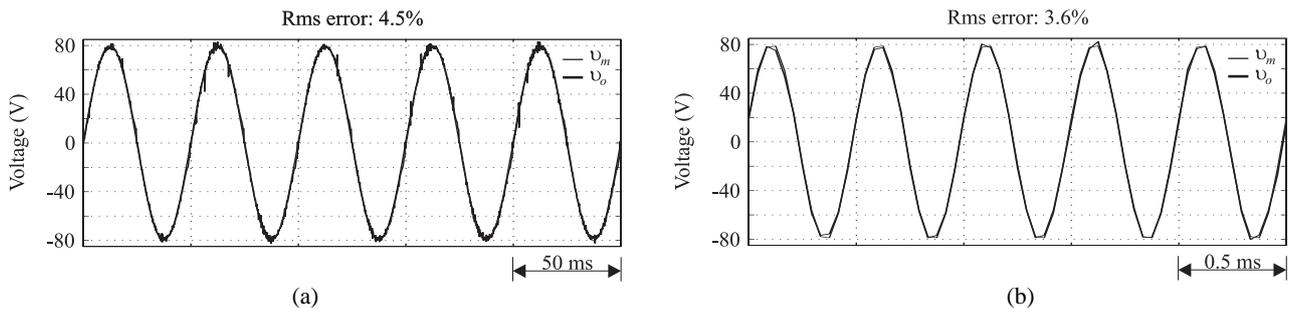


Fig. 8 Reference model output v_m and the measured voltage waveform v_o with the electrodynamic shaker. (a) 20 Hz. (b) 2 kHz.

By taking into account the meaningful changes that occur on the shaker dynamic behavior as function of the excitation frequency, the experimental results demonstrated the feasibility of this solution in controlling the armature voltage both at low and high frequencies and with fixed or swept frequency tests. In addition, it is important to note that, due to the reduced order model considered, the design of the robust adaptive controller requires only basic information about nominal electrical parameters of the shaker and the power amplifier.

V. CONCLUSION

The development of a high performance voltage controller for switching-mode AC power amplifiers intended to drive electrodynamic shakers is a very important issue. Low harmonic distortion and good reference tracking are required over a wide dynamic and frequency range. In addition, the system is strongly subjected to the effects of the back electromotive force and the parameters variations.

This paper has presented a sinusoidal voltage controller based on a robust model reference adaptive technique comprising reduced-order model of the shaker. The back electromotive force is dealt as unknown disturbance and the inductance of the armature is not modeled. The controller is digitally implemented and the design requires only basic information about nominal electrical parameters of the shaker and the power amplifier. Fixed and swept frequency experimental tests executed up to 2 kHz have demonstrated the high performance of this controller in adjusting the voltage of the shaker armature. Topics for future research include the development of an acceleration control scheme whose output command corresponds to the reference signal of the power amplifier.

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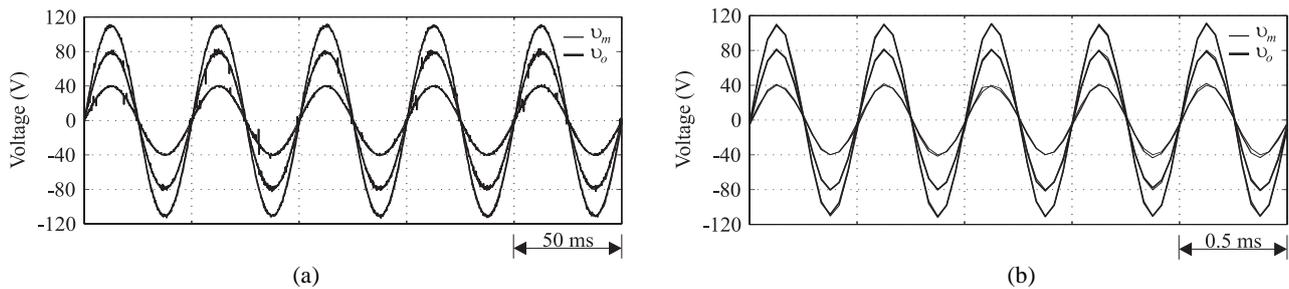


Fig. 9 Reference model output v_m and the measured voltage waveform v_o with different amplitudes. (a) 20 Hz. (b) 2 kHz.

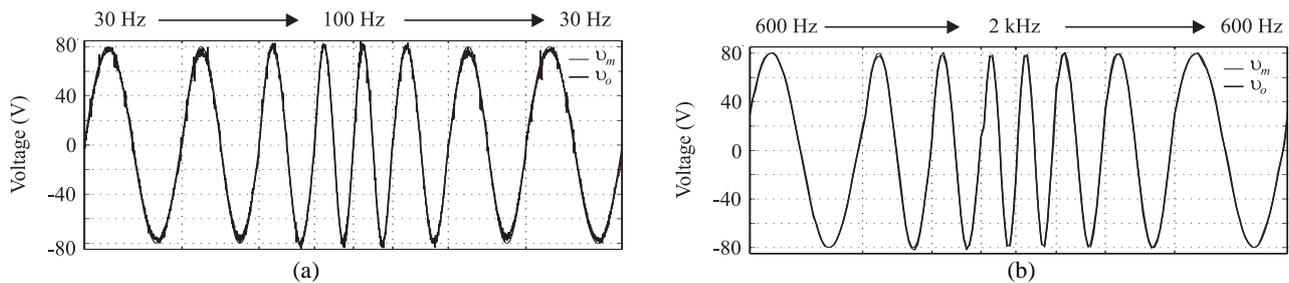


Fig. 10 Reference model output v_m and the measured voltage waveform v_o with swept frequency. (a) 30 to 100 Hz. (b) 600 Hz to 2 kHz.

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