Comparative Study of the Inductances of an Induction Motor with Rotor Eccentricities

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Abstract – In this work, the inductances of an induction motor, which rotor may be misaligned, are evaluated. Two types of analysis are performed: Analytical Method (AM) and Finite Element Method (FEM). For radial displacements, both methods show equivalent results but to evaluate inductances as a function of angular displacements, only with FEM was possible to get results as with AM it would be necessary intricate functions. The inductances obtained with FEM were included in simulations of a bearingless induction motor with split windings. This approach shows a good agreement with experimental results.

Keywords – Airgap Eccentricity, Bearingless Induction Motor, Finite Elements Method, Inductance.

I. INTRODUCTION

Induction motors with split windings operating as bearingless induction motor are being studied since 1989 [1], [2] and [3]. With this configuration it is possible to generate torque and radial positioning forces. With this approach some works were reported in the literature focusing on starting and constant speed operation [4], [5]. Strategies of speed control with good responses were also reported in [6]. Respect to the implementation of controllers, a significative progress has been reported, including the use of DSP’s. However, the effects of possible misalignment between rotor and stator were not considered.

With this in mind, the interaction between magnetic fluxes from induction motor when submitted to misalignment is being studied [7]-[8]. In [9], one interesting analytical method (AM) is presented. To study the effects of airgap misalignment, the surfaces of stator and rotor are approximated as cylinders.

In this work, the AM results are compared with simulations using the Finite Elements Method (FEM), which has been consolidated as an useful tool in the study of non-linear phenomena and of great complexity.

The aim is the study of the self and mutual inductances behavior of an induction motor when submitted to radial misalignment. These results will allow for the prediction of possible interferences of the magnetic fluxes inside the motor with the torque and radial forces. These concerns are present in the so called bearingless induction motor with split windings [10].

II. DETERMINATION OF MOTOR INDUCTANCES WITH NON UNIFORM AIRGAP

The machine analyzed in this work is a four poles, two phase induction motor, Figure 1. Each phase is composed of four windings that are connected in series in normal operation. When operating as a bearingless induction motor, each of the windings of one phase is supplied separately in order to control radial positioning. The other windings are connected in series and supplied simultaneously when is required the starting torque.

Table 1 shows the main parameters used to calculate inductances.

<table>
<thead>
<tr>
<th>Parameters of biphase induction motor</th>
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<tbody>
<tr>
<td>External diameter of stator</td>
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<tr>
<td>Internal diameter of stator</td>
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<tr>
<td>Rotor diameter</td>
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<tr>
<td>Airgap</td>
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<tr>
<td>Length of the rotor</td>
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<td>Section area of the stator slot</td>
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<tr>
<td>Area of the rotor slot</td>
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<tr>
<td>Number of wires in each slot of the stator</td>
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Fig. 1. Set of stator winding of the induction machine.
A. Analytical Method (AM)

Assuming that the eccentricity of the rotor is limited to the airgap, it is possible to approximate the inverse of the airgap as the following function [9]:

\[ P(\kappa) = \frac{1}{g(\kappa)} = \frac{1}{g_0} \sum_{i=0}^{\infty} P_i \cos \left( i(\kappa - \alpha) \right) \]  

(1)

where \( \kappa \) is the reference angle of stator windings, \( \alpha \) is the mechanical rotor displacement angle, \( g_0 \) is the uniform airgap, as shown in Figure 2, and

\[ P_0 = \frac{1}{\sqrt{1 - \varepsilon^2}} \]

\[ P_i = 2P_0 \left( \frac{1 - \sqrt{1 - \varepsilon^2}}{\varepsilon} \right)^i \]

(2)

where \( \varepsilon \) is the eccentricity between rotor and stator centers, calculated as:

\[ \varepsilon = \sqrt{X_0^2 + Y_0^2} / g_0 \]

(3)

Due to the magnetic field variation caused by the eccentricities of the airgap along any portion of winding \( v \), it is possible to find

\[ \int_{\alpha = \text{gap}}^{\beta = \text{gap}} H_v(\kappa) \cdot d\vec{i} = n_v(\kappa) i_v(\kappa) \]

(4)

where \( H_v \) is the magnetic field intensity, \( n_v \) is the turn function (number of coils), which depends of \( \kappa \), and \( i_v \) is the winding current.

Manipulating (4) and (1), using Gauss law for magnetic fields in closed surfaces, assuming that the magnetic permeability of the core is infinite, that magnetic field is homogenous along the axial direction, that the dispersion of the magnetic field on both extremities of the rotor is neglected and considering that the airgap is very small when compared to the rotor radius, self and mutual inductances can be evaluated as in (5) and (6) respectively

\[ L_{vv} = \frac{\lambda_{vv}}{i_v} = \mu_0 r \int_0^{2\pi} n_s(\kappa) N_v(\kappa) P(\kappa) d\kappa \]

(5)

\[ L_{vw} = \frac{\lambda_{vw}}{i_v} = \mu_0 r \int_0^{2\pi} n_s(\kappa) N_w(\kappa) P(\kappa) d\kappa \]

(6)

where \( N_v \) and \( N_w \) are the winding functions. For instance, for winding \( v \):

\[ N_v = n_v - \frac{\int_0^{2\pi} n_s(\kappa) P(\kappa) d\kappa}{\int_0^{2\pi} P(\kappa) d\kappa} \]

(7)

B. Finite Elements Method (FEM)

The complexity of the machine demands great care in its modeling through FEM. A three dimensional simulation would be more appropriate but it demands high computational effort. For this reason it was chosen a two dimensional simulation using an appropriated refinement on the mesh.

From electromagnetism the inductance can be expressed as:

\[ L = \frac{N}{I} \Psi \]

(8)

where \( L \) is the inductance, \( I \) is the current, \( N \) is the number of turns and \( \Psi \) is the magnetic flux which can be obtained from (9).

\[ \Psi = \int (\nabla \times A) \cdot ds \]

(9)

According to Stokes’s theorem and calculating the vector potential from its nodal values, it is possible to determine one relationship to calculate the inductance in a general form:

\[ L = \frac{N}{I} \sum_{s=t}^{m} \sum_{j=t}^{n} \left( S_j / n \right) \sum_{i=t}^{m} A_{ij} \]

(10)

where \( I \) is the length of the rotor, \( A_{ij} \) the nodal vector potential, \( S_t \) is the total area in study, \( S_j \) the element area, \( m \) is the number of elements and \( n \) is the number of nodes by element (first-order triangle elements).

III. COMPARATIVE RESULTS

Inductance established through FEM (10) depends directly on the magnetic vector potential, which changes according to the rotor position. According to the AM these depend on (1). In this work, self-inductances of phase A were evaluated using the AM and FEM. In Figures 3 and 4 are shown the self-inductances of windings A1 and A2 as a function of rotor horizontal displacements. The average errors are 8% approximately, which can be explained by the approximations used with the analytical method. This shows that FEM is a useful tool.

The results shown in Figures 5 and 6 would be difficult to be obtained with the analytical method. It is about the self-inductances of windings A1 and A2 when there are angular and radial displacements along the axis x. FEM shows that there is an important variation of self-inductances of winding A2 as the angular position of the rotor changes.
On the other hand, FEM shows that there is a strong influence of the angular displacement of the rotor in the mutual inductance between the stator and rotor, as seen in Figure 7.

IV. DYNAMIC SIMULATIONS

In order to verify the effect of inductances variations, as calculated by FEM, in simultaneous speed and position closed loop control systems, the behavior of the motor under dynamic conditions was simulated using the software Matlab™. Figure 8 shows the block diagram of a combined conventional indirect airgap field oriented vector control with the radial position control for the bearingless induction motor. In this case, the speed controller is a proportional integral (PI) type. The reference flux is held constant and the PID radial position controllers generate the $i_x$ and $i_y$ current components according to the error between the position references and the position sensor signals. Related to the motor model, an inductances matrix $L(\theta, x)$, composed by all the values obtained through the FEM, was included. The simulations consider two conditions. In the first one, the rotor is arbitrarily unaligned 0.2mm along the axis $x$ and the windings of phase A are activated in order to centralize the rotor. Details of the displacement control may be found in [11]. This process occurs during 0.6s. Figure 9 shows that the rotor is centralized in approximately 0.2s. On the other hand, Figure 10 shows the behavior of radial force, along the axis $x$. As noted, the force presents a ripple as the inductances are varying because of the rotor radial displacements. After 0.6s, the closed loop of speed vector control is initiated. Figures 9 and 10 show transient effects on both, radial displacement and force. Figure 11 shows the zoom of the rotor displacement. As observed, a vibration happens during all the starting. This result is in accordance with [10] and [12], where it was experimentally observed that the position controllers had not good performance when the motor was submitted to load torque. As seen in Figure 12, the net torque due to the vector control has a small ripple caused by the variations of inductances. This effect would not be foreseen if the analytical method to calculate the inductances were used. In Figure 13 the behavior of rotor speed after a step input of 900 rpm is shown.
Fig. 8. Block diagram of an indirect airgap field oriented vector control for a bearingless induction motor with split windings.

Fig. 9. Rotor radial displacement along the axis x.

Fig. 10. Radial force along the axis x.

Fig. 11. Zoom of rotor displacement.

Fig. 12. Torque during the motor starting.
In order to observe the effect of neglecting the inductance variations due to the angular displacements in the motor modeling, other simulations were performed. In this case, the inductance matrix was modified to set the inductance values as if the angular position were zero degrees so the inductances changes only by radial displacements.

Figures 14, 15 and 16 show the rotor displacement along the axis x, radial force and rotor speed, respectively. As seen, in the firsts 0.6s the behavior of rotor is the same of the previous case, because the same inductances were used. From 0.6s until 2s transient effects in the radial displacement and force are also observed. After 2s, the rotor suffers a constant and periodic vibration and the rotor reach a speed lower then the speed reference. Additionally, the simulation showed that the amplitude of stator current reached its maximum value, so this is the typical behavior of a detuned vector control. As seen in Figure 7, to neglect the dependence of angular displacement in mutual inductances between stator and rotor, even with the rotor centralized, implies an error into the value of the model parameters used in the vector control.

V. EXPERIMENTAL RESULTS

In order to test the complete control system shown in Figure 8, a workbench was implemented. It consists of a low power bearingless induction motor with nominal airgap of 1mm, a current power supply module with five CC-VSI converters, a target desktop computer with A/D and D/A cards and a host computer running in the LabView environment. The target computer runs in a real time operative system. In Figure 17, the picture of the motor is shown.
Using the same procedure as in the simulation, Figure 18 shows the behavior of rotor speed. After 1s, the closed loop speed control is activated and reaches the reference speed in approximately 8s.

The radial displacement of the rotor along the x-axis is shown in Figure 19. In this case, after the activation of the speed control, a strong vibration happens during 2s. As noted, if compared with Figures 9 or 11, the transient and the vibrations are similar but the amplitudes and durations are different. This can be explained if it is taken into account that in the simulations the current power sources are ideal and the controllers are continuous. In the actual implementation the sources are half-bridge inverters and the controllers are digitally implemented.

Simulations using the motor model with inductances calculated by FEM show good agreement with experimental results.

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REFERENCES


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