TWO-PHASE THREE-WIRE SHUNT ACTIVE POWER FILTER CONTROL
BY USING THE SINGLE-PHASE P–Q THEORY

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Abstract — This paper presents a study of a shunt active power filter (APF) to be applied to two-phase three-wire electric networks. The application of this new APF is presented and possible topologies are discussed. The control algorithm of the APF is based on the single-phase p–q theory, which is briefly reviewed. Then, alternative-α–β transformations for voltages and currents are proposed to implement the single-phase p–q calculations. These adaptations are followed by the description of the proposed current reference calculation algorithm. Digital simulation and experimental results validate the effectiveness of the proposed shunt APF and control algorithm in compensating current harmonics, reactive power and imbalances of two-phase three-wire loads, even under distorted mains voltage condition.


1. INTRODUCTION

In the last decades, the number and variety of appliances electronically controlled have drastically changed home and business environments of modern societies. Power electronics drives for heating, ventilation and air conditioning, washing machines, refrigerators, home running systems, electric stoves and cookers, microwave ovens and, more recently, wide screen LCD TV sets, notebooks, tablets, smart phones, compact fluorescent lamps or led based lighting systems are examples of these devices.

Besides of the increase of electricity consumption, a great number of these new loads are fed by power electronic converters draining non-sinusoidal currents from the electric mains. These distorted currents cause power quality problems such as voltage distortion, electromagnetic interference and malfunction of communication and instrumentation equipment, overload and overheating of electric distribution feeders and transformers, among others [1]–[3].

Passive, active and hybrid power filters are power conditioning devices to be connected in parallel and in series with the electrical network. They are designed to filter harmonics, improving power quality indexes [4]–[6].

The principles of active power filters (APF) were proposed at the ending of 70's and in the beginning of 80's [7], [8]. Different converter topologies and control algorithms have been widely studied and reported in technical literature for shunt and series APF [4], [6], [9].

Shunt APF are power electronics converters specially designed and controlled to synthesize harmonic currents on its output terminals to compensate distorted currents drained by nonlinear loads. They can also be controlled to compensate reactive power at the load terminals or demanded by the power system [10].

Despite the number of works about three–phase APF, efforts were also carried on to develop active filters to mitigate power quality problems in single-phase electrical systems [11], [12]. In this specific type of applications, the single-phase APF is installed at low voltages networks to compensate harmonic currents generated by single-phase loads [4], [11]–[14].

However, the application of APF to compensate two-phase three-wire electric networks (2 phases and neutral) has not been enough explored and reported in the literature. These systems represent a significant portion of residential and commercial electrical installations and, probably due to the cost of the active power filter structure, two-phase APF have been employed to solve problems caused by specific loads like welding machines [15] and ac electrified railways [16].

A potential field for the application of two–phase three–wire shunt APF (2φ3w–APF) in residential and commercial electricity networks could become real by the use of on-board converters of the electric vehicles as recently proposed in [12], [17]. Since vehicle-to-grid connections are usually made to one or two phases of the power distribution system, this extra functionality could provide a cost reduction when it is compared to the separated implementation of the shunt APF. It can also represent an extra income to the vehicle owner since an ancillary service will be provided to the electric utility system. In this future scenario, the massive application of active power filters on residential and commercial installations can be a way to mitigate power quality problems in distribution networks, minimizing reactive and harmonic power flows, reducing losses and enhancing the capability of active power transfer through the electric feeders.

In this context, this paper presents an algorithm to calculate the reference currents used to control a two-phase three-wire shunt APF. The proposed algorithm is based on the p–q theory definitions for single-phase electrical networks. Section II will show and discuss three converter topologies for the 2φ3w–APF. A brief review of the p–q theory and its adaptation to single-phase networks will be presented in Section III. Section IV describes the algorithm of reference current calculation to control the 2φ3w–APF. Simulation and experimental results are presented in Sections V and VI.
respectively. Finally, Section VII presents the conclusions of this work.

II. SYSTEM DESCRIPTION

Figure 1 shows the block diagram of a typical residential consumer fed by two phases and a neutral wire, where \( L_s \) is the inductance of each supply feeder. This supply system is derived from the conventional three-phase four-wire ac system. Inside the load block there are line-to-neutral and line-to-line loads. The shunt 2φ3w–APF is connected to the point of common coupling (PCC) through three inductances \( L_f \) to limit the APF current ripple [18].

Using the control algorithm that will be shown in this work, based on the sinusoidal compensation strategy, the currents \( i_{fa}, i_{fb} \) and \( i_{fn} \) synthesized by the 2φ3w–APF will compensate all the reactive power and the harmonic currents drained by the load. After the compensation, the mains currents become sinusoidal and without any phase delay with respect to each system phase voltage [10].

Figure 2 shows three converter topologies suitable to built the 2φ3w–APF [15]. Despite the independent control of the compensating current of each phase, the two single-phase converters of Figure 2.a have the higher number of active switches. They share the same dc capacitor whose average voltage should be at least 3/2 times higher than the ac phase peak voltage [19], [20].

On the other hand, since the two single-phase converters have one of their output terminals connected to the same conductor of the mains, it is possible to share their functionality. This characteristic allows to use the converter shown in the Figure 2.b to synthesize the 2φ3w–APF. This topology has the advantage of using two less active switches. In this case, the dc capacitor voltage should be adjusted to be higher than 3/2 times of the peak value of the ac line voltage [19], [20].

The last option is the converter shown in Figure 2.c. It has the advantage of using the lowest number of semiconductor switches. However the neutral wire should be connected to the midpoint between the two dc capacitors. Thus, each capacitor dc voltage should be setup to be at least equal to 3/2 times of the peak value of the ac line voltage. One disadvantage of this topology is the necessity of equalization of the capacitors dc voltages to avoid unsymmetrical output voltages at the ac terminals of the converter [19], [20].

It is also possible to perform an harmonic analysis for the previous converter topologies. The converter of Figure 2.a can be controlled to generate a unipolar output voltage waveform while the only option to modulate the converter of Figure 2.c is by using a bipolar pattern. This characteristic will result in a lower harmonic content for the first topology when it is compared with the last one. For the topology of Figure 2.b, the two phases of the converter can be switched in a synchronous way with the third branch to produce unipolar and bipolar waveforms [21].

Therefore, based on above considerations the topology of Fig. 2.b was chosen to build the 2φ3w–APF proposed in this work. In the next section it will be presented some concepts about the single–phase \( p–q \) theory and its adaptation to calculate reference compensating currents for the shunt 2φ3w–APF.

III. BRIEF REVIEW OF THE \( P–Q \) THEORY

The instantaneous power theory, or the \( p–q \) theory, has been firstly presented in a japanese conference as a set of power definitions based on instantaneous values of voltages and currents. Since there is no restriction to be imposed
to the voltages and currents, such definitions are valid for systems containing harmonics or not, and during transients or steady-state conditions. These power definitions are specially useful to calculate reference signals for APFs controllers [10].

Originally developed for three-phase three-wire systems, the p–q theory uses the Clarke Transformation to map instantaneous values of voltages and currents from the natural a-b-c system coordinates into an orthogonal α/β-0 reference frame coordinate.

According to [10] and considering the system voltages \((v_α, v_β, v_0)\) and currents \((i_α, i_β, i_0)\), in the new reference frame coordinate, it is possible to calculate instantaneous values of real \(p\), imaginary \(q\) and zero-sequence \(p_0\) powers as shown bellow:

\[
\begin{bmatrix}
  p \\
p \\
q
\end{bmatrix}
= 
\begin{bmatrix}
  v_α & v_β & 0 \\
v_β & -v_α & 0 \\
0 & 0 & v_0
\end{bmatrix}
\begin{bmatrix}
i_α \\
i_β \\
i_0
\end{bmatrix},
\]  

where \(p\) and \(q\) are the fundamental frequency components of the single-phase system voltage, already represented in the \(αβ\) reference frame.

\[i_f^* = \frac{v_α}{v_α^2 + v_β^2} p_0 - \frac{v_β}{v_α^2 + v_β^2} q_0 \]  

where \(i_f^*\) is the reference current for the shunt \(1φ\)-APF.

As explained before, if the \(1φ\)-APF is controlled to compensate all current harmonics and the full reactive power at the load terminals, then, the compensating powers in (3) should be chosen as \(p_c = p\) and \(q_c = q + \tilde{q}\).

To successfully implement the sinusoidal control strategy, it is necessary to extract the fundamental component of mains voltage in case it is distorted [10], [14]. In this case the reference current for the APF should be calculated by

\[i_f^* = \left(\frac{v_α}{v_α^2 + v_β^2}\right) \tilde{p} - \left(\frac{v_β}{v_α^2 + v_β^2}\right) q_0\]  

Or, by using the indirect method as follows:

\[i_f^* = i_α - \left(\frac{v_α}{v_α^2 + v_β^2}\right) \tilde{p}\]  

where \(v_α\) and \(v_β\) are the fundamental frequency components of the single-phase system voltage, already represented in the \(αβ\) reference frame.

IV. CONTROL ALGORITHM OF THE 2ϕ3w–APF

The algorithm proposed here treats the 2ϕ3w–APF as two independent single-phase APFs with a shared branch which is connected to the mains neutral conductor.

However, before the aforementioned single-phase p–q theory can be applied to calculate the reference signals for the APF some adaptations are proposed. The quadrature filter \(H(s)\) is tuned to shift the fundamental frequency component phase by \(π/2\) rad. This expected behaviour will not happen for the harmonics, which will be shifted by different phase angles.

Figure 3 shows an example where a current waveform is transformed into the \(αβ\) coordinates using the filter function given in (2). The usage of the currents waveforms shown in Figure 3 will cause malcalculations of instantaneous powers and then the control algorithm of the APF will generate wrong reference compensating signals.

\[H(s) = \frac{1 - sT}{1 + sT}\]  

where \(T = 1/ω\) and \(ω\) is the system angular fundamental frequency.

Considering the system voltage and current written in the \(αβ\) reference frame, it is possible to use (1) to calculate the instantaneous powers \(p\) and \(q\) related to the single-phase system. Then, the reference current for the single-phase shunt APF can be calculated by using (1) on its inverse form. The proper selection of power parcels that should be compensated results in

\[i_f^* = \frac{v_α}{v_α^2 + v_β^2} p_0 - \frac{v_β}{v_α^2 + v_β^2} q_0\]  

where \(v_α\) and \(v_β\) are the fundamental frequency components of the single-phase system voltage, already represented in the \(αβ\) reference frame.

Fig. 3. Current waveforms obtained with \(H(s)\).

To overcome the previous problem it will be used an alternative–\(αβ\) transformation. The method consists of defining \(i_β\) equal to the measured phase current. The other current \(i_α\) is generated by using a buffer that stores a quarter of the cycle of the fundamental frequency of the current \(i_β\). Figure 4 shows the result obtained by using the time delay buffer. The shifted signal is an exact copy of the original waveform. The choice of buffer length, for a given sampling
frequency, will guarantee the $\pi/2$ rad phase shift for the measured signal.

In the same way as for the current, the voltage $v_\beta$ must be defined equal to the measured phase voltage. However, instead of generating a quadrature signal $v_\alpha$ by a buffer, this paper proposes the usage of a second-order generalized integrator (SOGI) similar to that shown in Figure 5 [25].

The use of the SOGI presents some advantages if it is compared with the conventional method (quadrature filter or buffer plus single-phase PLL). Firstly, its design and implementation are quite simple and well described in literature [25]. It does not require buffers to store the measured voltages, diminishing memory usage in the case of digital implementation. It also attenuates the harmonics and generates a quadrature signal simultaneously, improving the performance of the control algorithm. Thus, SOGI presents the desired capability of filtering and quadrature signal generation in an easily-implementable block structure.

In the $2\phi3w$ system shown in Figure 1, the proposed $\alpha/\beta$ transformation must be applied to four quantities: $v_a$, $v_b$, $i_{La}$, and $i_{Lb}$, before the single-phase $p-q$ theory can be used. Thus, by applying (1) for each separate phase, four power components are calculated: $p_a$, $p_b$, $q_a$, and $q_b$, where the subscripts denote each system phase.

The average and oscillatory components of $p_a$ and $p_b$ have to be separated (using a LPF, for example). The average two-phase active power is then defined by

$$\bar{p}_{2\phi} = \bar{p}_a + \bar{p}_b.$$  \hfill (6)

The power components given in (6) will be useful when the $2\phi3w$–APF should compensate load unbalances. According to the proposed adaptations, the currents to be synthesized by each phase of the shunt active filter are given by

$$i_{fx}^* = \left( \frac{v_{x\beta}}{v_{x\alpha}^2 + v_{x\beta}^2} \right) \left[ p_x - \left( \frac{p_{dc} + \bar{p}_{2\phi}}{2} \right) - \left( \frac{v_{x\alpha}}{v_{x\alpha}^2 + v_{x\beta}^2} \right) q_x \right]$$ \hfill (7)

where the subscript $x \in \{a, b\}$ and $p_{dc}$ represents an average amount of active power that the $2\phi3w$–APF should drain or inject into the mains to regulate the VSC dc-capacitor voltage [10].

The third reference current, that is the current that will be synthesized on the neutral conductor, must be calculated by

$$i_{fn}^* = -(i_{fa}^* + i_{fb}^*).$$ \hfill (8)

Figure 6 shows the block diagram of the control algorithm of the $2\phi3w$–APF. Inside each block, it is indicated the number of the equation used as it appears in the text. Two low-pass filters (LPF) with 10 Hz of cut-off frequency separate the average parts of the instantaneous powers $p_a$ and $p_b$. An additional loop, with a PI controller, was included in the block diagram to generate the power $p_{dc}$ used to regulate the VSC dc voltage. Three hysteresis current loops guarantee that the compensation currents ($i_{fa}^*$, $i_{fb}^*$ and $i_{fn}^*$) synthesized by the APF track the reference currents ($i_{fa}^*$, $i_{fb}^*$ and $i_{fn}^*$) calculated by (7) and (8), respectively.

The power components given in (6) will be useful when the $2\phi3w$–APF should compensate load unbalances. According to the proposed adaptations, the currents to be synthesized by each phase of the shunt active filter are given by
V. SIMULATION RESULTS

A digital simulation has been carried out to validate the proposed adaptations and the method to generate reference currents for the $2\phi3w$–APF. Figure 7 shows the system configuration used to carry out the digital simulation studies. The control algorithm blocks, shown Figure 6, and the current and voltage measuring sensors, as well as their feedback control loops, were not drawn in Figure 7 in order to make it clearer. All the setup system parameters are given in Table I.

At a first glance the reader may question the value of the system voltage shown in Table I. However the electric network shown in Figure 7 was built in the laboratory of the Power Electronics and Automation Group (NAEP) to investigate the impact of the charging and discharging process of the batteries of a small EV (electric crosskart). So, this choice would allow the comparison of the results obtained with the digital and experimental models of the $2\phi3w$–APF.

The phase voltages and the load currents at the PCC are measured and sampled with a frequency of 21 kHz. The signals of each phase are transformed into their average and oscillating parts and then are separated into their average and oscillating parts and then used to calculate the reference currents for each phase of the $2\phi3w$–APF.

Different types of current controllers can be employed to force the inverter output currents to track their references $i_{fa}, i_{fb}$ and $i_{fc}$ [26]. Since the main goal of this work is the control algorithm of the $2\phi3w$–APF, an hysteresis controller was used for each phase of the VSC. Despite its variable switching frequency, this controller is easily implemented and it guarantees a very small amplitude error or phase delay for each of the fundamental frequency components before they are used to calculate the synthesized current.

The SOGI parameters were designed to be equal to $k = 0.35$ and $\omega_r = 377$ rad/s. The value of $k$ was chosen to guarantee a good harmonic rejection while the resonance frequency $\omega_r$ was made equal to the fundamental angular frequency of the system [25]. Two buffers, with 87 memory positions each, are used to carry out the $\alpha\beta$ transformations of phase currents as explained in Section IV. They were dimensioned to store 1/4 of the fundamental frequency current waveforms sampled with 21 kHz.

The PI controller used to regulate the VSC dc–voltage was designed with a proportional gain and a time constant equal to 20 W/V and 2 W/(V.s), respectively. These values were chosen to assure a zero steady state error and to block the 120 Hz ripple common in single–phase applications. The

![Fig. 7. Setup system for digital simulation studies and experimental validation.](image-url)
presence of this ripple would cause distortion in the reference currents.

Figures 8 and 9 show phase voltages measured at PCC and the SOGI outputs signals for phases $a$ and $b$, respectively. Note that there is a high frequency ripple in the phase voltages due to the switching operation of the VSC. This ripple could be minimized if a LC or LCL filter was connected at the VSC output terminals.

Figures 10 and 11 show load currents of phases $a$ and $b$, respectively. Each figure shows the measured current and its $\alpha\beta$ components. It can be noted that $\alpha$ components are exact copies of the measured currents, but shifted by $1/4$ of period.

Figure 12 shows the neutral current flowing between the mains and the load in the $2\phi 3w$-system. This current is the result of the sum of $i_{la}$ and $i_{lb}$.

The results presented in this section validate the proposed control algorithm. They also confirm that the compensating currents are balanced in a $2\phi 3w$-system. In the next section it will be presented the results obtained with the experimental implementation of such system.

VI. EXPERIMENTAL RESULTS

As explained before, the electric network shown in Figure 7 was built in laboratory to investigate the impact of
the connection of a small EV. In this scenario, the embedded converter of a small EV (electric kart cross) had its internal controller changed to operate as a $2\phi 3w$–APF. The VSC uses three IGBT modules (SKS200B6CI73V03) manufactured by Semikron. The whole control algorithm for the $2\phi 3w$–APF was implemented in a floating point digital signal processor (DSP) TMS320F28335 of Texas Instruments.

The system phase voltages and the currents drained by the load in phase $a$, phase $b$ and neutral are shown in Figures 16, 17 and 18, respectively. Note that $i_{la}$ presents higher amplitude and distortions when it is compared to $i_{lb}$. The neutral current is also highly distorted. The current in phase $a$ is also phase delayed with respect to the phase voltage.

By performing the proposed control strategy, the $2\phi 3w$–shunt APF injects the current waveforms shown in Figure 19 into the PCC phases $a$, $b$ and neutral conductor, respectively. The compensated current waveforms, that is, those drained from the two-phase voltage source phases $a$, $b$ and neutral are respectively presented in Figures 20, 21 and 22.

Despite the distorted voltage at PCC, the compensated currents are almost sinusoidal. This behavior is explained by the operation of the SOGIs. They extract the fundamental component of each system phase voltage before they are used by the control algorithm. As observed in the digital simulation results there are high order harmonics in system voltages due to the switching operation of the VSC. Also, these currents

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**Fig. 16.** Experimental mains voltage and load current - phase $a$.

**Fig. 17.** Experimental mains voltage and load current - phase $b$.

**Fig. 18.** Experimental load current - neutral.

**Fig. 19.** Experimental $2\phi 3w$–shunt APF synthesized currents.

**Fig. 20.** Experimental voltage and source current - phase $a$.

**Fig. 21.** Experimental voltage and source current - phase $b$. 
are in phase with fundamental phase voltages. That is result of the adopted sinusoidal control strategy and it means the power delivered by the mains is mainly active power in the fundamental frequency. Additionally, these results show the effectiveness of the 2φ3w–APF in compensating load imbalances, since currents in phases \( a \) and \( b \) have nearly the same amplitude.

The obtained results confirmed the simulation results, allowing a two-phase mains to feed a non-linear, reactive and imbalanced load with sinusoidal currents with the same amplitude. The comparison of currents THD between load and source showed quantitatively the effectiveness of the two-phase shunt APF.

Future works shall investigate the performance of the proposed algorithm under system frequency variation conditions. Different compensations strategies, such as constant power strategy, may be also studied and useful depending on the application desired to the proposed shunt APF.

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![Fig. 22. Experimental source current - neutral.](image-url)


### BIOGRAPHIES

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