Induction motor transient analysis in the ABC/abc reference frame: application to the modeling and simulation of soft-starters

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***Abstract* - This work describes a framework for transient analysis of induction motors, composed by a computational environment and an experimental arrangement, directed for both educational and research purposes. Machine modelling is fully based on ABC/abc induction machine model, without requiring knowledge about any kind of reference frame transformation. An analytical method for the inductance matrix inversion is presented. One application refers to the modeling and simulation of soft-starters, which are validated by experimental measurements.**

I. INTRODUCTION

The study of induction machine transients is usually done with the DQ reference frame, which requires previous knowledge of the "generalized machine theory" [1]. This may be further complicated by the use of the PU system, which requires some familiarity and a careful choice of the base system.

An interesting survey of induction machine models [2], supported by EPRI, showed the convenience of the use of the ABC/abc model when power electronics systems were applied either at the stator or at the machine rotor. This paper stated:

"*The inherent defect of the direct application of the ABC/abc model for digital simulation of the motor transients is the large computation time required for the inversion of the time-varying inductance matrix during each step of integration. Many other models and approaches were developed to avoid this time consuming operation. One straightforward approach is ... to invert the inductance matrix analytically before numerical integration of the equations. In general, such a procedure is extremely difficult.*"

Under certain conditions it is possible to obtain analytical expressions, as done in [2]. These expressions will not be reproduced due to their complexity. This paper proposes an improved method (Eqs. 17 and 18) for the inductance matrix inversion, which reduces computation time dramatically, simplifying thus the use of the ABC/abc model and eliminating the aforementioned "inherent defect". A previous paper from the same authors [3] presented a simplified approach for the inductance matrix inversion. In this paper we also apply the proposed method to the modelling and simulation of soft-starters.

II. ABC/abc INDUCTION MACHINE MODEL

Basic equations of the machine (assuming sinusoidal MMF and neglegible saturation and losses in the core) are:

$$[V] = [R]\times[I] + \frac{d[\lambda]}{dt} \quad \textbf{(1)}$$

$$[\lambda] = [L]\times[I] \quad \textbf{(2)}$$

and:

$[V]=[v_A \;\; v_B \;\; v_C \;\; v_a \;\; v_b \;\; v_c]^T$ are the voltages applied to each stator and rotor phases;

$[I]=[i_A \;\; i_B \;\; i_C \;\; i_a \;\; i_b \;\; i_c]^T$ are the currents in stator and rotor phases;

$[\lambda]=[\lambda_A \;\; \lambda_B \;\; \lambda_C \;\; \lambda_a \;\; \lambda_b \;\; \lambda_c]^T$ are the fluxes linked with each stator and rotor phases.

From (1) and (2) results:

$$\frac{d[I]}{dt} = [L]^{-1}\times\left\{-[R] - \frac{d[L]}{dt}\right\}\times[I] + [L]^{-1}\times|V| \quad \textbf{(3)}$$

Most terms of the inductance matrix are functions of rotor position θ. Taking this into account:

$$\frac{d[L]}{dt} = \frac{d[L]}{d\theta}\times\frac{d\theta}{dt} = \frac{d[L]}{d\theta}\times\omega_R \quad \textbf{(4)}$$

where ω_R is the rotor speed. Grouping (3) and (4) results:

$$\frac{d[I]}{dt} = [L]^{-1}\times\left\{-[R] - \omega_R\frac{d[L]}{d\theta}\right\}\times[I] + [L]^{-1}\times|V| \quad \textbf{(5)}$$

Using the usual form of control systems, we can state:

$$\frac{d[I]}{dt} = [A]\times[I] + [B]\times[V] \quad \textbf{(6)}$$

Where $[A]$ and $[B]$ matrices can be identified in (5):

$$[A] = [L]^{-1}\times\left\{-[R] - \omega_R\frac{d[L]}{d\theta}\right\} \quad \textbf{(7)}$$

$$[B] = [L]^{-1} \quad \textbf{(8)}$$

The main problem is to establish the inductance matrix and improve its inversion. Matrix $[L]$ is given by:

$$\begin{bmatrix} (L_{lS}+M_S) & -0.5M_S & -0.5M_S & M_{SR}f_1 & M_{SR}f_2 & M_{SR}f_3 \\ -0.5M_S & (L_{lS}+M_S) & -0.5M_S & M_{SR}f_3 & M_{SR}f_1 & M_{SR}f_2 \\ -0.5M_S & -0.5M_S & (L_{lS}+M_S) & M_{SR}f_2 & M_{SR}f_3 & M_{SR}f_1 \\ M_{SR}f_1 & M_{SR}f_3 & M_{SR}f_2 & (L_{lR}+M_R) & -0.5M_R & -0.5M_R \\ M_{SR}f_2 & M_{SR}f_1 & M_{SR}f_3 & -0.5M_R & (L_{lR}+M_R) & -0.5M_R \\ M_{SR}f_3 & M_{SR}f_2 & M_{SR}f_1 & -0.5M_R & -0.5M_R & (L_{lR}+M_R) \end{bmatrix} \quad \textbf{(9)}$$

Where: L_{lS} = stator leakage inductance [H]
M_S = stator mutual inductance [H]
L_{lR} = rotor leakage inductance [H]
M_R = rotor mutual inductance [H]
M_{SR} = M_{RS} = stator-rotor mutual inductance [H]

and

$$f_1 = \cos(\theta) \qquad f_2 = \cos(\theta + 2\pi/3) \qquad f_3 = \cos(\theta - 2\pi/3) \quad \textbf{(10)}$$

where θ is the stator-rotor magnetic angle.

If stator and rotor have equal number of turns (or referring rotor elements to stator, or using pu units) then:

$$M_S = M_R = M_{SR} \qquad \textbf{(11)}$$

If we take into account that:

$$i_A + i_B + i_C = 0 \qquad and \qquad i_a + i_b + i_c = 0 \qquad \textbf{(12)}$$

the inductance matrix $[L]$ can be written as:

$$[L] = \begin{bmatrix} L_S & 0 & 0 & M_{SR} f_1 & M_{SR} f_2 & M_{SR} f_3 \\ 0 & L_S & 0 & M_{SR} f_3 & M_{SR} f_1 & M_{SR} f_2 \\ 0 & 0 & L_S & M_{SR} f_2 & M_{SR} f_3 & M_{SR} f_1 \\ M_{SR} f_1 & M_{SR} f_3 & M_{SR} f_2 & L_R & 0 & 0 \\ M_{SR} f_2 & M_{SR} f_1 & M_{SR} f_3 & 0 & L_R & 0 \\ M_{SR} f_3 & M_{SR} f_2 & M_{SR} f_1 & 0 & 0 & L_R \end{bmatrix} \qquad \textbf{(13)}$$

where:

$$L_S = L_{lS} + \frac{3}{2} M_S \qquad L_R = L_{lR} + \frac{3}{2} M_R \qquad \textbf{(14)}$$

All these inductances are easily measurable in a YY connected slip ring induction machine. In a normal cage machine, these inductances can be estimated from the no-load and locked-rotor essays.

From (13) results:

$$\frac{d[L]}{dt} = -\omega_R M_{SR} \begin{bmatrix} 0 & 0 & 0 & g_1 & g_2 & g_3 \\ 0 & 0 & 0 & g_3 & g_1 & g_2 \\ 0 & 0 & 0 & g_2 & g_3 & g_1 \\ g_1 & g_3 & g_2 & 0 & 0 & 0 \\ g_2 & g_1 & g_3 & 0 & 0 & 0 \\ g_3 & g_2 & g_1 & 0 & 0 & 0 \end{bmatrix} \qquad \textbf{(15)}$$

where

$$g_1 = \sin(\theta) \qquad g_2 = \sin(\theta + 2\pi/3) \qquad g_3 = \sin(\theta - 2\pi/3) \qquad \textbf{(16)}$$

III. INDUCTANCE MATRIX ANALYTICAL INVERSION

If an electrical transient simulation is done directly from (6), the main problem will be the inversion of the inductance matrix $[L]$ at each simulation step, although this can be done using modern hardware and software tools. One approach is to apply the Cholesky decomposition since the inductance matrix is "symmetric positive definite".

Pillay [2] showed that the inverse matrix can be expressed analytically, but a slightly different and much simpler set of expressions can be used. We first calculate 4 auxiliary coefficients:

$$K_1 = \frac{L_S L_R}{M_{SR}^2} \qquad K_2 = \frac{K_1 - 3/4}{K_1 - 9/4}$$
$$K_3 = \frac{-3/4}{K_1 - 9/4} \qquad K_4 = \frac{-1/M_{SR}}{K_1 - 9/4} \qquad \textbf{(17)}$$

and then all other elements of the inverse matrix:

$$[L]^{-1} = \begin{bmatrix} \frac{K_2}{L_S} & \frac{K_3}{L_S} & \frac{K_3}{L_S} & K_4 f_1 & K_4 f_2 & K_4 f_3 \\ \frac{K_3}{L_S} & \frac{K_2}{L_S} & \frac{K_3}{L_S} & K_4 f_3 & K_4 f_1 & K_4 f_2 \\ \frac{K_3}{L_S} & \frac{K_3}{L_S} & \frac{K_2}{L_S} & K_4 f_2 & K_4 f_3 & K_4 f_1 \\ K_4 f_1 & K_4 f_3 & K_4 f_2 & \frac{K_2}{L_R} & \frac{K_3}{L_R} & \frac{K_3}{L_R} \\ K_4 f_2 & K_4 f_1 & K_4 f_3 & \frac{K_3}{L_R} & \frac{K_2}{L_R} & \frac{K_3}{L_R} \\ K_4 f_3 & K_4 f_2 & K_4 f_1 & \frac{K_3}{L_R} & \frac{K_3}{L_R} & \frac{K_2}{L_R} \end{bmatrix} \qquad \textbf{(18)}$$

The coefficients f_1 f_2 f_3 were already defined in (10).

Electromagnetic torque is calculated from:

$$T_E = -\frac{P}{2} M_{SR} \Big[+ (i_A i_a + i_B i_b + i_C i_c) \sin(\theta) \ldots + (i_A i_b + i_B i_c + i_C i_a) \sin(\theta + 2\pi/3) \ldots + (i_A i_c + i_B i_a + i_C i_b) \sin(\theta - 2\pi/3) \Big] \qquad \textbf{(19)}$$

Speed is given by:

$$\frac{d\omega_R}{dt} = \frac{T_E - T_{Load}}{J} \qquad \textbf{(20)}$$

The relations (6), (19) and (20) compose a set of 8 differential equations that can be solved using Simulink, by which induction machine transients can be simulated, in a very fast and accurate way.

One further improvement, almost obvious, is that only the evaluation of 2 stator and 2 rotor currents is necessary, reducing the system to 6 differential equations.

A second improvement is to consider fluxes as state variables (instead of currents). In this case, the fundamental equation is derived from (1):

$$\frac{d[\lambda]}{dt} = [V] - [R] \times [I] \qquad \textbf{(21)}$$

After each integration step currents are recalculated from:

$$[I] = [L]^{-1} \times [\lambda] \qquad \textbf{(22)}$$

using (18). The main advantage is that matrix $[A]$ doesn't have to be calculated.

The use of fluxes as state variables can be applied to simulate induction machine transients on sinusoidal supplies (balanced or unbalanced) and to simulate induction machines driven by voltage source PWM inverters.

The use of fluxes as state variables is somewhat difficult in some systems where power electronics is combined with induction machines, resulting in current constrained conditions (discontinous current conditions). Soft starters are a good example of these systems.

IV. EXPERIMENTAL ARRANGEMENT

An experimental arrangement was built around a 3 HP/220V/60 Hz induction machine that can be driven directly

from the main, by soft-starters or by several different PWM inverters. Load torque and inertia can be adjusted. Torque measurements are obtained from a rotating torque transducer and a mechanical balance system. Direct access to rotor variables is possible since this is a slip-ring machine, which also has special coils for evaluation of DQ stator flux. A multichannel data acquisition system stores all relevant variables.

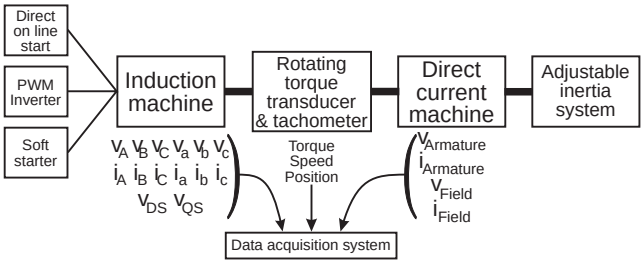

Fig. 1 Experimental setup

All inductance parameters are obtained from voltage and current measurements exciting stator coils, producing Fig. 2 (left) or rotor coils, Fig. 2 (right). Considerable variation in inductances due to saturation can be observed making measurements at several voltage/flux levels (not shown here due to space limitations). Detailed information about the induction machine is given in Appendix I.

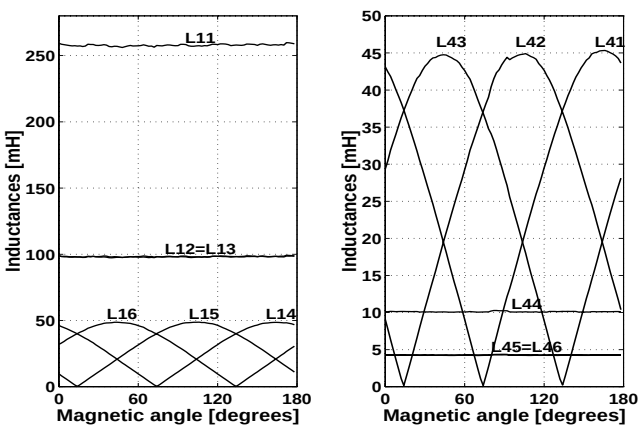

Fig. 2 Inductance measurements when exciting the stator (left) and rotor (right). Indexes 1,2,3 refer to stator and 4,5,6 refer to rotor coils. Results from 360 measurements.

V. SIMULATION AND EXPERIMENTAL RESULTS

Preliminary results are shown below. Mismatches between experimental and simulation results require further analysis and the use of better parameters. All simulations are performed with the fluxes as state variables, as explained in Section IV.

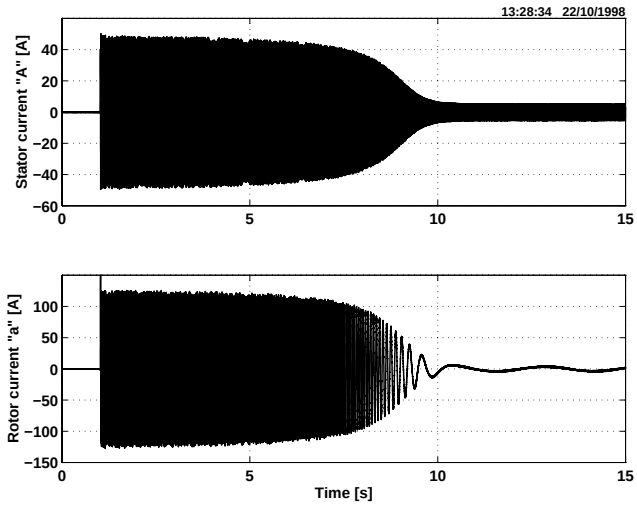

Fig. 3 Stator current (upper trace) and rotor current (lower trace) during direct on-line start. Experimental.

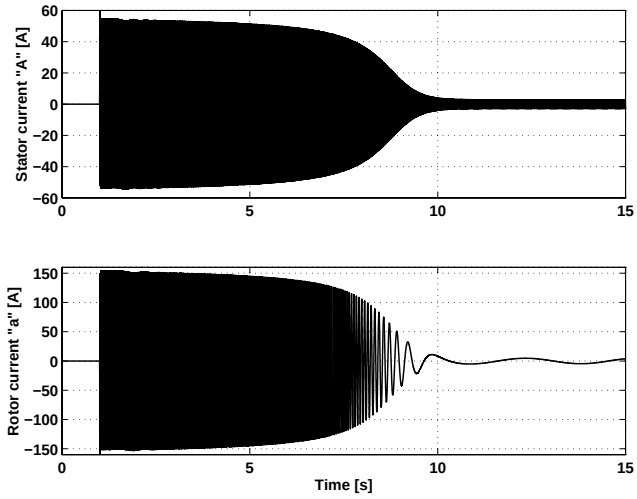

Fig. 4 Stator current (upper trace) and rotor current (lower trace) during direct on-line start. Simulation.

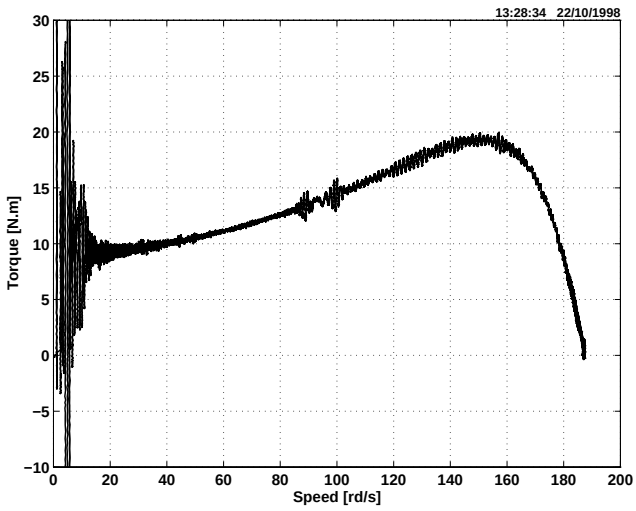

Fig. 5 Dynamic torque × speed curve during direct on-line start. Experimental.

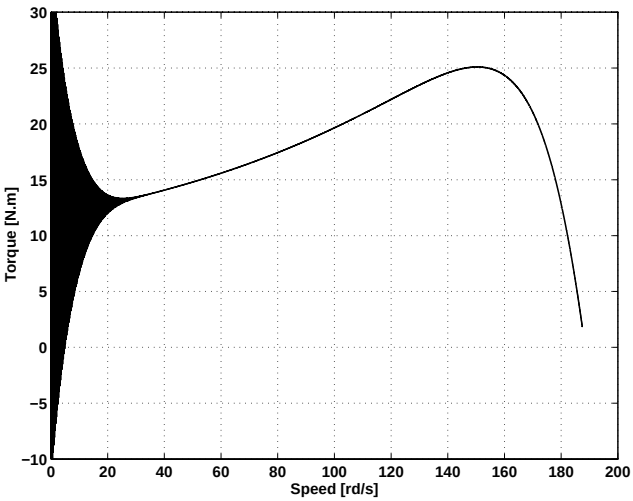

Fig. 6 Dynamic torque x speed curve during direct on-line start. Simulation.

VI. SOFT-STARTERS

The simulation of soft-starters has been the object of several papers [4, 5, 6, 7]. Soft-starters can be easily simulated with the proposed induction machine model (using currents as state variables). Figure 7 shows a soft starter diagram, where one can identify the main voltage and current variables:

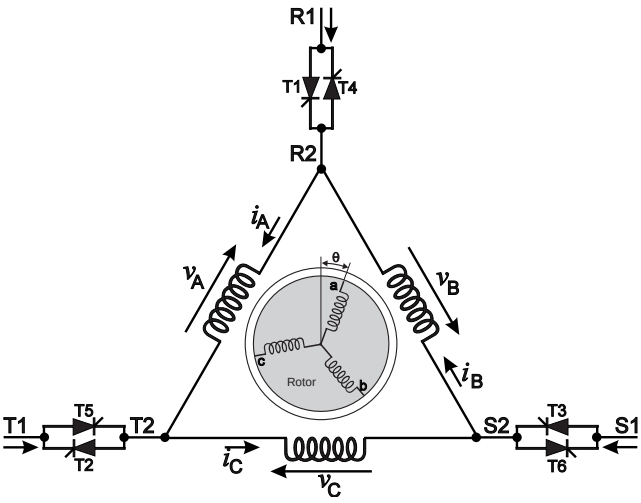

Fig. 7 Soft-starter diagram.

For dynamic simulation purposes, there are three basic topologies to be considered:

3 thyristors conduct simultaneously
2 thyristors conduct simultaneously
0 thyristors conduct simultaneously

These topologies combine in two different modes. In the continuous current conduction mode 3 and 2 topologies alternate. In the discontinuous current conduction mode 2 and 0 topologies succeed.

Modeling of 3 thyristor conduction topology is done applying directly (21).

Modeling of 2 thyristor conduction topology requires a little more attention. Consider the situation where only phases R1 and T1 conduct as shown in Fig. 8:

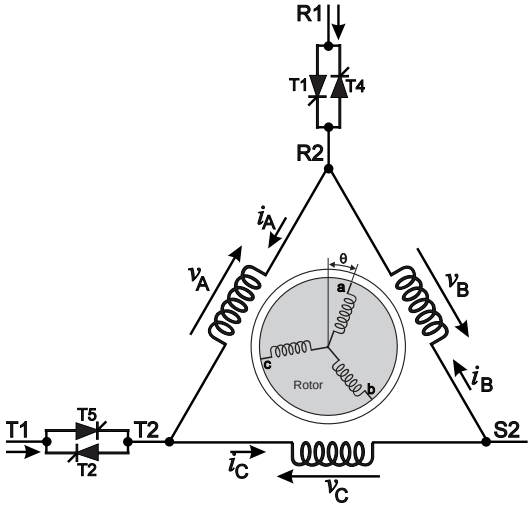

Fig. 8 Soft-starter diagram when only phases R1 and T1 conduct.

The only stator equation to be considered refers to phase ***A***. All other stator currents can be calculated using:

$$i_B = i_C \tag{23}$$

$$i_A + i_B + i_C = 0 \tag{24}$$

From (23) and (24) results:

$$i_B = i_C = -\frac{i_A}{2} \tag{25}$$

Since it is sufficient to calculate stator current I_A and two of the rotor currents, it is possible to modify (21) resulting:

$$\begin{aligned} \frac{d\lambda_A}{dt} &= v_A - R_S i_A \\ \frac{d\lambda_a}{dt} &= -R_R i_a \\ \frac{d\lambda_b}{dt} &= -R_R i_b \end{aligned} \tag{26}$$

The relationship between fluxes and currents in this topology is given by an auxiliary inductance matrix $[L_{Aux2}]$, derived from (2) and (13):

$$\begin{bmatrix} \lambda_A \\ \lambda_a \\ \lambda_b \end{bmatrix} = [L_{Aux2}] \times \begin{bmatrix} i_A \\ i_a \\ i_b \end{bmatrix} \tag{27}$$

where:

$$[L_{Aux2}] = \begin{bmatrix} L_S & M_{SR}(f_1 - f_3) & M_{SR}(f_2 - f_3) \\ M_{SR}\left(f_1 - \frac{f_3}{2} - \frac{f_2}{2}\right) & L_R & 0 \\ M_{SR}\left(f_2 - \frac{f_1}{2} - \frac{f_3}{2}\right) & 0 & L_R \end{bmatrix} \tag{28}$$

Calculation of currents from fluxes requires the inversion of $[L_{Aux2}]$ at each simulation step.

Modeling of the 0 thyristor conduction topology is very simple, since (21) reduces to (there are no conducting phases in the stator and only 2 rotor currents must be calculated):

$$\frac{d\lambda_a}{dt} = -R_R i_a \qquad \frac{d\lambda_b}{dt} = -R_R i_b \qquad \textbf{(29)}$$

The relationship between fluxes and currents in this topology is derived directly from (2) and (13), since all motor currents are known.

The soft-starter and induction motor simulation program is programmed as a Simulink block. Computational details will not be covered in this paper but further information can be obtained with the authors.

Due to the lack of space, only basic results will be presented, concerning the two operating modes (continuous and discontinuous current conduction), both at locked rotor conditions.

Experimental results for continuous current mode condition are given in Figs. 9 and 10. The corresponding simulation results are presented in Figs. 11 and 12. Simulation is **very** sensitive to the measured machine inductances, which are affected by saturation. In order to show this simulations with 2 inductance sets are presented, one measured at 220 V_{RMS} (inductance set A) and the other (inductance set B) at 88 V_{RMS} (the voltage level established by the soft-starter), which produces much better agreement with experimental results.

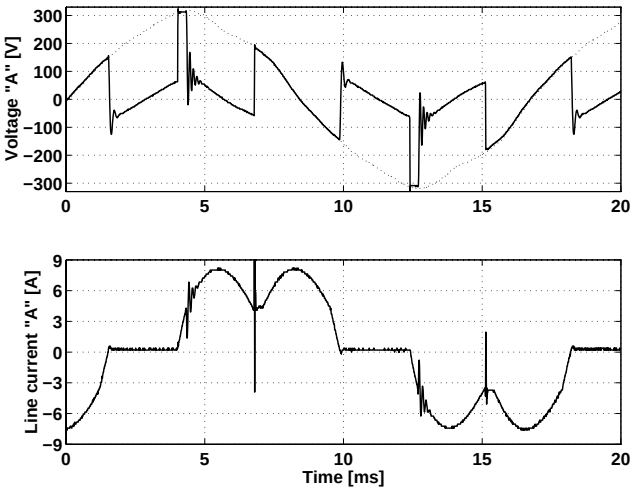

Fig. 9 Stator voltage and stator line current in the continuous current conduction mode (experimental).

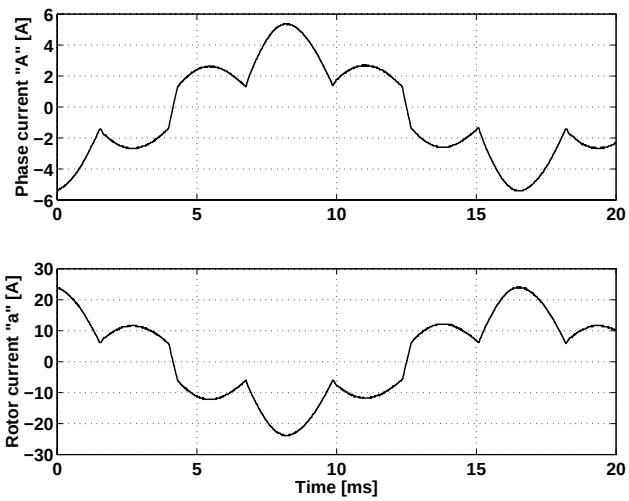

Fig. 10 Stator phase current and rotor current in the continuous current conduction mode (experimental).

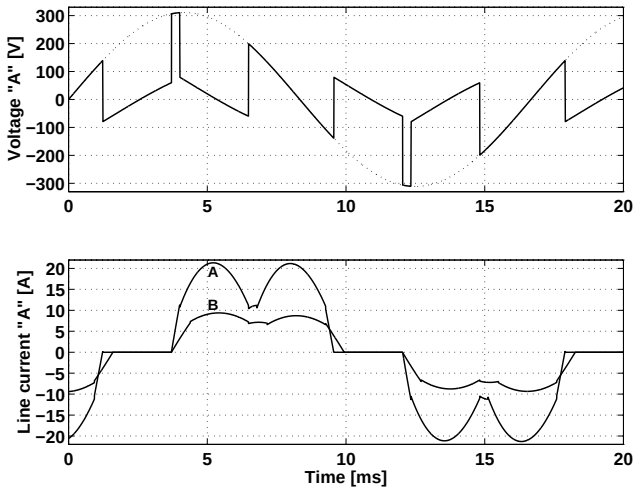

Fig. 11 Stator voltage and stator line current in the continuous current conduction mode (simulation).

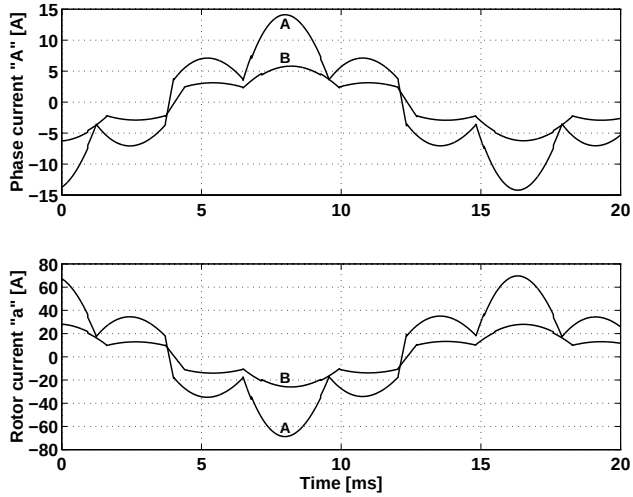

Fig. 12 Stator phase current and rotor current in the continuous current conduction mode (simulation).

Experimental results for discontinuous current mode condition are given in Figs. 13 and 14. The corresponding simulation results are presented in Figs. 15 and 16.

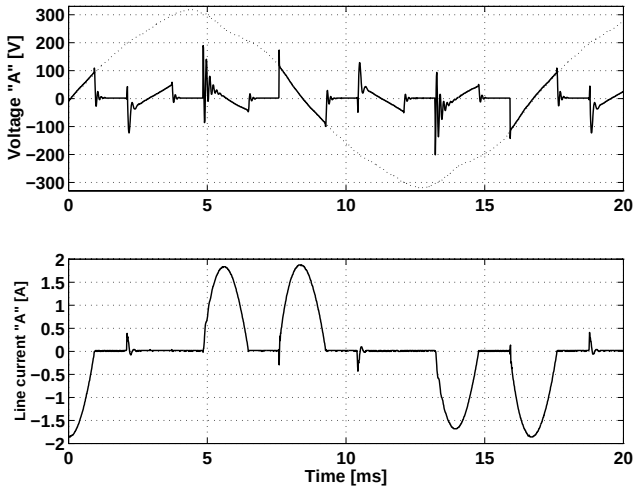

Fig. 13 Stator voltage and stator line current in the discontinuous current conduction mode (experimental).

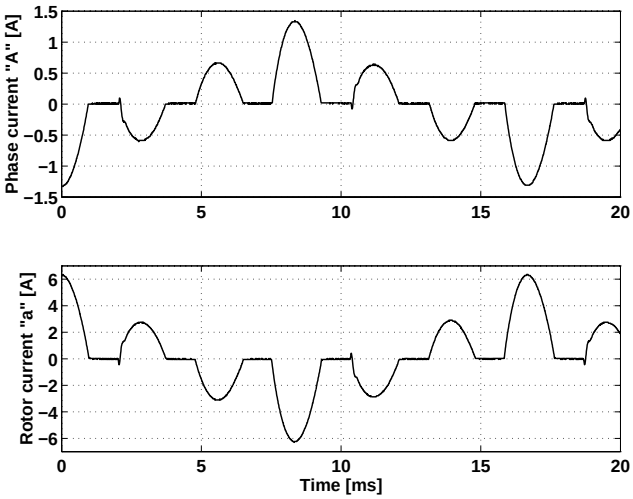

Fig. 14 Stator phase current and rotor current in the discontinuous current conduction mode (experimental).

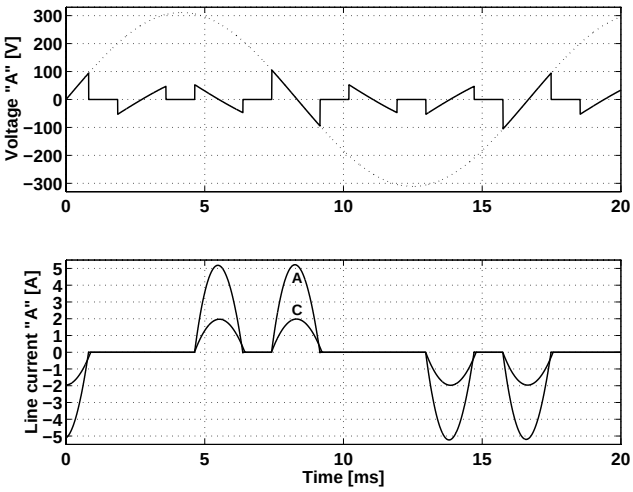

Fig. 15 Stator voltage and stator line current in the discontinuous current conduction mode (simulation).

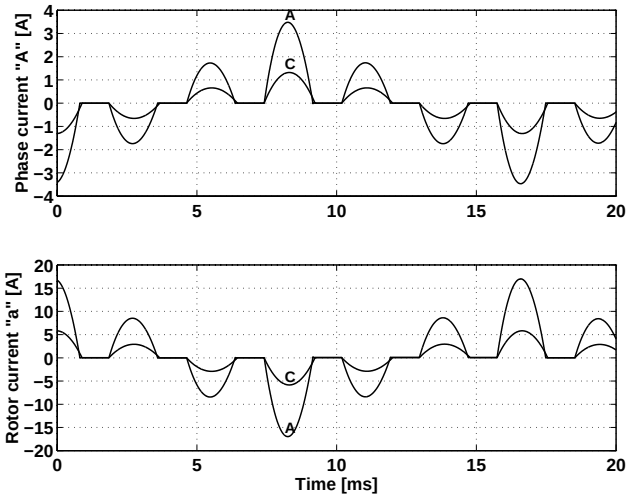

Fig. 16 Stator phase current and rotor current in the discontinuous current conduction mode (simulation).

Once again, simulation results are **very** sensitive to the measured machine inductances. To show this simulations with 2 inductance sets are presented, one measured at 220 V_{RMS} (inductance set A) and the other (inductance set C) at 28 V_{RMS} (the voltage level established by the soft-starter), which produces better agreement with experimental results. It is clear evident that special care must be taken in the measurement of inductance values in order to assure agreement between experimental and simulation results.

VI. CONCLUSIONS

An improved analytical method for the inversion of the inductance matrix was proposed. This method simplifies the use of the ABC/abc induction machine model. One important application is for the teaching of induction machine transients in undergraduate courses, without requiring from students any previous knowledge about "generalized machines theory". Other applications involve the modeling and simulation of power electronics systems such as soft-starters. Experimental results validate the proposed method. Special care must be taken with inductance values used since simulation results are very sensitive to these parameters.

VII. REFERENCES

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Appendix I - MACHINE PARAMETERS

Nominal power	2240 W
Nominal speed	1700 RPM
Nominal frequency	60 Hz
Number of poles	4
Nominal torque	12.58 N.m
Nominal stator voltage	220 V (line)
Nominal stator current	9 A Connection : Δ
Nominal rotor current	27 A Connection : Y
Stator resistance	r_S = 1.79Ω
Rotor resistance	r_R = 0.0442 Ω
Stator leakage inductance	L_{lS} = 61.48 mH
Stator mutual inductance	M_S = 196.37 mH
Rotor leakage inductance	L_{lR} = 1.59 mH
Rotor mutual inductance	M_R = 8.52 mH
Stator-rotor mutual inductance	M_{SR} = 46.83 mH
Total system inertia	J = 1.4 kg.m2

Set	V_{RMS}	L_S [mH]	L_R [mH]	M_{SR} [mH]	K_1
A	220	356.04	14.37	46.83	2.33
B	88	438.0	19.6	58.5	2.51
C	28	410.0	18.96	56.1	2.47