Quasi Time-Invariant Parameter Modeling of Unbalanced Induction Machines

C.B. Jacobina[1], J.E. Chaves Fl.[2], A.M.N. Lima[1]
[1]Laboratório de Eletrônica Industrial e Acionamento de Máquinas
Departamento de Engenharia Elétrica, Universidade Federal da Paraíba
Caixa Postal 10105, 58109-970 Campina Grande, PB, Brasil
Fax: ++55(83)310-1015 Email: jacobina@dee.ufpb.br
[2]Departamento de Eletricidade, Universidade Federal do Amazonas - AM - Brasil

I. ABSTRACT

This paper proposes a quasi time–invariant model to characterize three-phase unbalanced *ac* machine. The primary three-phase machine model is transformed into a modified *odq* representation that take in account the number of coil turns of each the stator winding. Except for a very small term depending of the machine resistance, the *odq* model can be considered as similar to an *odq* balanced machine model. The vector model is suitable for deriving control schemes for unbalanced operation and useful for computer simulation studies as will be presented in the paper.

II. INTRODUCTION

It is well known that induction motors are widely used in several types of industries. Consequently, failures of such motors should degrade the performance of the industry by stopping the production flow resulting and in some cases causing life risks like the fault of a cooling system in a nuclear power plant. Then, it is very important to detect incipient failures and to take the necessary safety precautions. The most frequent failures in inductions motors occurs with the coils of the stator windings.

There is a relatively large number of papers that discuss the operation of induction motors under fault conditions [1], [2], [3], [4], [5]. However, none of works have been addressed the use of coordinate transformations to obtain a simplified, useful and elegant model for describing the behavior of the induction motor under fault conditions.

The main objective of this paper is to propose a modified *odq* coordinate transformation to simplify the representation of the dynamic behavior of an induction motor when there exist failures in the coils of the stator windings. The proposed model can also be extended to study failures that occur in the coils of the rotor windings in the case of wound-rotor induction motor.

III. THREE-PHASE MACHINE

A. Three-Phase Model

Fig. 1 shows a circuit representation of a three-phase machine. In the equations given below for the three-phase machine, the superscripts s and r indicate the stator and the rotor reference frame, respectively.

$$\mathbf{v}^s_{s123} = \mathbf{r}_{s123}\mathbf{i}^s_{s123} + \frac{d\boldsymbol{\phi}^s_{s123}}{dt} \tag{1}$$

$$\mathbf{v}^r_{r123} = \mathbf{r}_{r123}\mathbf{i}^r_{r123} + \frac{d\boldsymbol{\phi}^r_{r123}}{dt} \tag{2}$$

$$\boldsymbol{\phi}^s_{s123} = \mathbf{l}_{s123}\mathbf{i}^s_{s123} + \mathbf{l}_{sr123}\mathbf{i}^r_{r123} \tag{3}$$

$$\boldsymbol{\phi}^s_{r123} = \mathbf{l}_{r123}\mathbf{i}^r_{r123} + \mathbf{l}^T_{sr123}\mathbf{i}^s_{s123} \tag{4}$$

$$T_e = p(\mathbf{i}^s_{s123})^T \frac{\partial \mathbf{l}_{sr123}}{\partial \theta_r}\mathbf{i}^r_{r123} \tag{5}$$

In the above equations, $\mathbf{v}^s_{s123}$, $\mathbf{i}^s_{s123}$, $\boldsymbol{\phi}^s_{s123}$, $\mathbf{v}^r_{r123}$, $\mathbf{i}^r_{r123}$, $\boldsymbol{\phi}^r_{r123}$ are column vectors representing stator voltages, stator currents, stator fluxes, rotor voltages, rotor currents and rotor fluxes, respectively. The column vector representing the stator voltages is defined as $\mathbf{v}^s_{s123} = [\mathbf{v}^s_{s1}\ \mathbf{v}^s_{s2}\ \mathbf{v}^s_{s3}]^T$, while the other vectors are defined similarly. Also, T_e, T_m, θ_r and p represent the electromagnetic torque, load torque, shaft position and the number of poles, respectively.

To complete the model given by equations (1)-(5) it is necessary to define the matrices $\mathbf{r}_{s123}$, $\mathbf{r}_{r123}$, $\mathbf{l}_{s123}$ and $\mathbf{l}_{r123}$ that represent the resistances and inductances of the stator and rotor respectively.

It is important to remark that all the hypothesis assumed in the development of the standard Park's model are also considered in this case. The basic difference of the proposed approach with respect to the standard modelling is that the number of coil turns of each winding appears explicitly in the inductance as well as in the resistance matrices. In this way any failure related with the coils of the stator and rotor can be represented by changing the number of turns in the model equations. In this case the stator inductance matrix $\mathbf{l}_{s123}$ is given by:

$$\mathbf{l}_{s123} = \begin{bmatrix} P_1 n_{s1}^2 & P_2 n_{s1} n_{s2} & P_2 n_{s1} n_{s3} \\ P_2 n_{s1} n_{s2} & P_1 n_{s2}^2 & P_2 n_{s2} n_{s3} \\ P_2 n_{s1} n_{s3} & P_2 n_{s2} n_{s3} & P_1 n_{s3}^2 \end{bmatrix}$$

or by

$$\mathbf{l}_{s123} = \begin{bmatrix} P_1 n_{s1} n_n & P_2 n_{s1} n_n & P_2 n_{s1} n_n \\ P_2 n_{s2} n_n & P_1 n_{s2} n_n & P_2 n_{s2} n_n \\ P_2 n_{s3} n_n & P_2 n_{s3} n_n & P_1 n_{s3} n_n \end{bmatrix} \mathbf{N}_s = \mathbf{l}'_{s123}\mathbf{N}_s$$

where

$$\mathbf{N}_s = \begin{bmatrix} n_{s1}/n_n & 0 & 0 \\ 0 & n_{s2}/n_n & 0 \\ 0 & 0 & n_{s3}/n_n \end{bmatrix}$$

and $P_1 = P_{\sigma s} + P_{ms}$, $P_2 = -P_{ms}/2$ where $P_{\sigma s}$, P_{ms} denote permeances of the respective magnetic circuits while n_{s1}, n_{s2}, n_{s3} and n_n are the number of turns of each stator winding and a normalizing number of turns respectively.

Following the same reasoning, the matrix corresponding to the mutual inductance between stator and rotor $\mathbf{l}_{sr123}$ is given by:

$$\mathbf{l}_{sr123} = \begin{bmatrix} P_3 n_{s1} n_{r1} a_1 & -P_3 n_{s1} n_{r2} a_2 & -P_3 n_{s1} n_{r3} a_3 \\ -P_3 n_{s2} n_{r1} a_3 & P_3 n_{s2} n_{r2} a_1 & -P_3 n_{s2} n_{r3} a_2 \\ -P_3 n_{s3} n_{r1} a_2 & -P_3 n_{s3} n_{r2} a_3 & P_3 n_{s3} n_{r3} a_1 \end{bmatrix}$$

or by

$$\begin{aligned} \mathbf{l}_{sr123} &= \begin{bmatrix} P_3 n_{s1} n_n a_1 & -P_3 n_{s1} n_n a_2 & -P_3 n_{s1} n_n a_3 \\ -P_3 n_{s2} n_n a_3 & P_3 n_{s2} n_n a_1 & -P_{3s2} n_n a_2 \\ -P_3 n_{s3} n_n a_2 & -P_3 n_{s3} n_n a_3 & P_3 n_{s3} n_n a_1 \end{bmatrix} \\ &= \mathbf{l}'_{sr123} \mathbf{N}_r \end{aligned}$$

where P_3 is the permeance between a stator phase and a rotor phase when $\theta = 0$;

$$\begin{aligned} a_1 &= \cos(\theta) \\ a_2 &= \cos(\theta + 2\pi/3) \\ a_3 &= \cos(\theta - 2\pi/3) \end{aligned}$$

and

$$\mathbf{N}_r = \begin{bmatrix} n_{r1}/n_n & 0 & 0 \\ 0 & n_{r2}/n_n & 0 \\ 0 & 0 & n_{r3}/n_n \end{bmatrix}$$

where n_{r1}, n_{r2}, n_{r3} denote the number of turns of each rotor winding.

Similarly, the matrix corresponding to the rotor inductance matrix $\mathbf{l}_{r123}$ is

$$\mathbf{l}_{r123} = \begin{bmatrix} P_4 n_{r1}^2 & P_5 n_{r1} n_{r2} & P_5 n_{r1} n_{r3} \\ P_5 n_{r1} n_{r2} & P_4 n_{r3}^2 & P_5 n_{r2} n_{r3} \\ P_5 n_{r1} n_{r3} & P_5 n_{r2} n_{r3} & P_4 n_{r3}^2 \end{bmatrix}$$

or

$$\mathbf{l}_{r123} = \begin{bmatrix} P_4 n_{r1} n_n & P_5 n_{r1} n_n & P_5 n_{r1} n_n \\ P_5 n_{r2} n_n & P_4 n_{r2} n_n & P_5 n_{r2} n_n \\ P_5 n_{r3} n_n & P_5 n_{r3} n_n & P_4 n_{r3} n_n \end{bmatrix} = \mathbf{l}'_{r123} \mathbf{N}_r$$

where $P_4 = P_{\sigma r} + P_{mr}$, $P_5 = -P_{mr}/2$

The resistances of each stator and rotor windings are also assumed to be dependent on the respective number of coil turns. Then, the stator and rotor resistance matrices are given by:

$$\mathbf{r}_{s123} = \begin{bmatrix} r'_{s1} n_{s1}^2 & 0 & 0 \\ 0 & (r''_{s2} + r'_{s1} n_{s2}) n_{s2} & 0 \\ 0 & 0 & (r''_{s3} + r'_{s1} n_{s3}) n_{s3} \end{bmatrix}$$

or by

$$\mathbf{r}_{s123} = \mathbf{r}''_{s123} \mathbf{N}_s + \mathbf{r}'_{s123} \mathbf{N}_s$$

and by

$$\mathbf{r}_{r123} = \begin{bmatrix} r'_{r1} n_{r1}^2 & 0 & 0 \\ 0 & (r''_{r2} + r'_{r1} n_{r2}) n_{r2} & 0 \\ 0 & 0 & (r''_{r3} + r'_{r1} n_{r3}) n_{r3} \end{bmatrix}$$

or by

$$\mathbf{r}_{r123} = \mathbf{r}''_{r123} \mathbf{N}_r + \mathbf{r}'_{r123} \mathbf{N}_r$$

B. Modified odq model

If the standard *odq* transformation is directly applied in the case $n_{s1} \neq n_{s2} \neq n_{s3}$ and $n_{r1} \neq n_{r2} \neq n_{r3}$ the resulting *odq* model will have time-varying parameters that depend on the rotor angular position and on the *dq* position angle. A time-varying *odq* model is very hard to work with and has the same complexity of the to primitive three-phase representation. However, a quasi-time invariant model can be obtained if the matrices N_s and N_r are adequately used as shown in the following:

Introducing these matrices in the equations (1)-(5) and defining $\mathbf{v}_s^{s\prime} = \mathbf{N}_s^{-1} \mathbf{v}_s^s$, $\boldsymbol{\phi}_s^{s\prime} = \mathbf{N}_s^{-1} \boldsymbol{\phi}_s^s$, $\mathbf{i}_s^{s\prime} = \mathbf{N}_s \mathbf{i}_s^s$, $\mathbf{v}_r^{s\prime} = \mathbf{N}_r^{-1} \mathbf{v}_r^s$, $\boldsymbol{\phi}_r^{s\prime} = \mathbf{N}_r^{-1} \boldsymbol{\phi}_r^s$ and $\mathbf{i}_r^{s\prime} = \mathbf{N}_r \mathbf{i}_r^s$ it can be shown that:

$$\mathbf{v}_{s123}^{s\prime} = \mathbf{N}_s^{-1} \mathbf{r}''_{s123} \mathbf{i}_{s123}^{s\prime} + \mathbf{N}_s^{-1} \mathbf{r}'_{s123} \mathbf{i}_{s123}^{s\prime} + \frac{d\boldsymbol{\phi}_{s123}^{s\prime}}{dt} \quad (6)$$

$$\mathbf{v}_{r123}^{r\prime} = \mathbf{N}_r^{-1} \mathbf{r}''_{r123} \mathbf{i}_{r123}^{r\prime} + \mathbf{N}_r^{-1} \mathbf{r}'_{r123} \mathbf{i}_{r123}^{r\prime} + \frac{d\boldsymbol{\phi}_{r123}^{r\prime}}{dt} \quad (7)$$

$$\boldsymbol{\phi}_{s123}^{s\prime} = \mathbf{N}_s^{-1} \mathbf{l}'_{s123} \mathbf{i}_{s123}^{s\prime} + \mathbf{N}_s^{-1} \mathbf{l}'_{sr123} \mathbf{i}_{r123}^{r\prime} \quad (8)$$

$$\boldsymbol{\phi}_{r123}^{r\prime} = \mathbf{N}_r^{-1} \mathbf{l}'_{r123} \mathbf{i}_{r123}^{r\prime} + \mathbf{N}_r^{-1} \mathbf{l}'^T_{sr123} \mathbf{i}_{s123}^{s\prime} \quad (9)$$

$$T_e = p(\mathbf{i}_{s123}^{s\prime})^T \frac{\partial \mathbf{l}'_{sr123}}{\partial \theta_r} \mathbf{i}_{r123}^{r\prime} \quad (10)$$

The next step in the modelling is to use the $123 - odq$ coordinate transformation matrix defined by:

$$\mathbf{P}(\alpha) = \sqrt{2/3} \begin{bmatrix} 1/\sqrt{2} & \cos(\alpha) & -\sin(\alpha) \\ 1/\sqrt{2} & \cos(\alpha - 2\pi/3) & -\sin(\alpha - 2\pi/3) \\ 1/\sqrt{2} & \cos(\alpha + 2\pi/3) & -\sin(\alpha + 2\pi/3) \end{bmatrix}$$

and consequently

$$\mathbf{x}_{s123}^{s\prime} = \mathbf{P}_s \mathbf{x}_{sodq}^{a\prime}$$

$$\mathbf{x}_{r123}^{r\prime} = \mathbf{P}_r \mathbf{x}_{rodq}^{a\prime}$$

with $\mathbf{P}_s = \mathbf{P}(\delta)$ and $\mathbf{P}_r = \mathbf{P}(\delta - \theta_r)$ where δ is angular position of the *dq* axis. Applying these transformations to the three-phase equations yields:

$$\begin{aligned} \mathbf{v}_{sodq}^{a\prime} &= \mathbf{P}_s^{-1} \mathbf{N}_s^{-1} \mathbf{r}''_{s123} \mathbf{P}_s \mathbf{i}_{sodq}^{a\prime} + \mathbf{P}_s^{-1} \mathbf{N}_s^{-1} \mathbf{r}'_{s123} \mathbf{P}_s \mathbf{i}_{sodq}^{a\prime} \\ &\quad + \frac{d\boldsymbol{\phi}_{sodq}^{a\prime}}{dt} + \omega_a \mathbf{P}_s^{-1} \frac{\partial \mathbf{P}_s}{\partial \delta} \boldsymbol{\phi}_{sodq}^{a\prime} \end{aligned} \quad (11)$$

$$\mathbf{v}^{a\prime}_{rodq} = \mathbf{P}_r^{-1}\mathbf{N}_r^{-1}\mathbf{r}''_{r123}\mathbf{P}_r\mathbf{i}^{a\prime}_{rodq} + \mathbf{P}_r^{-1}\mathbf{N}_r^{-1}\mathbf{r}'_{r123}\mathbf{P}_r\mathbf{i}^{a\prime}_{rodq} + \frac{d\boldsymbol{\phi}^{a\prime}_{rodq}}{dt} + (\omega_a - \omega_r)\mathbf{P}_r^{-1}\frac{\partial \mathbf{P}_r}{\partial(\delta-\theta)}\boldsymbol{\phi}^{a\prime}_{rodq} \quad (12)$$

$$\boldsymbol{\phi}^{a\prime}_{sodq} = \mathbf{P}_s^{-1}\mathbf{N}_s^{-1}\mathbf{l}'_{s123}\mathbf{P}_s\mathbf{i}^{a\prime}_{sodq} + \mathbf{P}_s^{-1}\mathbf{N}_s^{-1}\mathbf{l}'_{sr123}\mathbf{P}_r\mathbf{i}^{a\prime}_{rodq} \quad (13)$$

$$\boldsymbol{\phi}^{a\prime}_{rodq} = \mathbf{P}_r^{-1}\mathbf{N}_r^{-1}\mathbf{l}'_{r123}\mathbf{P}_r\mathbf{i}^{a}_{rodq} + \mathbf{P}_r^{-1}\mathbf{N}_r^{-1}\mathbf{l}'^{T}_{sr123}\mathbf{P}_s\mathbf{i}^{a\prime}_{sodq} \quad (14)$$

$$T_e = p(\mathbf{i}^{s\prime}_{sodq})^T\mathbf{P}_s^{-1}\frac{\partial \mathbf{l}'_{srodq}}{\partial\theta_r}\mathbf{P}_r\mathbf{i}^{r\prime}_{rodq} \quad (15)$$

where $\omega_r = d\theta_r/dt$ is the shaft speed and $\omega_a = d\delta_r/dt$ is the speed of the dq axis.

After some algebraic manipulations it follows that:

$$\mathbf{v}^{a/}_{sodq} = \boldsymbol{\varepsilon}^a_s + \mathbf{r}'_{sodq}\mathbf{i}^{a\prime}_{sodq} + \frac{d\boldsymbol{\phi}^{a\prime}_{sodq}}{dt} + \omega_a\mathbf{J}\boldsymbol{\phi}^{a\prime}_{sodq} \quad (16)$$

$$\mathbf{v}^{a/}_{rodq} = \boldsymbol{\varepsilon}^a_r + \mathbf{r}'_{rodq}\mathbf{i}^{a\prime}_{rodq} + \frac{d\boldsymbol{\phi}^{a\prime}_{rodq}}{dt} + (\omega_a - \omega_r)\mathbf{J}\boldsymbol{\phi}^{a\prime}_{rodq} \quad (17)$$

$$\boldsymbol{\phi}^{a\prime}_{sodq} = \mathbf{l}'_{sodq}\mathbf{i}^{a\prime}_{sodq} + \mathbf{l}'_{srodq}\mathbf{i}^{a\prime}_{rodq} \quad (18)$$

$$\boldsymbol{\phi}^{a\prime}_{rodq} = \mathbf{l}'_{rodq}\mathbf{i}^{a}_{rodq} + \mathbf{l}'^{T}_{srodq}\mathbf{i}^{a\prime}_{sodq} \quad (19)$$

$$T_e = p\frac{3}{2}n_r^2P_{msr}(i^a_{sq}i^a_{rd} - i^a_{sd}i^a_{rq}) \quad (20)$$

where

$$\boldsymbol{\varepsilon}^a_s = \mathbf{r}''_{sodq}\mathbf{i}^{a\prime}_{sodq}$$

$$\boldsymbol{\varepsilon}^a_r = \mathbf{r}''_{rodq}\mathbf{i}^{a\prime}_{r123}$$

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{l}''_{sodq} = n_n^2\begin{bmatrix} P_{\sigma s} & 0 & 0 \\ 0 & P_{\sigma s} + (3/2)P_{ms} & 0 \\ 0 & 0 & P_{\sigma s} + (3/2)P_{ms} \end{bmatrix}$$

$$\mathbf{l}'_{rodq} = n_n^2\begin{bmatrix} P_{\sigma r} & 0 & 0 \\ 0 & P_{\sigma r} + (3/2)P_{mr} & 0 \\ 0 & 0 & P_{\sigma r} + (3/2)P_{mr} \end{bmatrix}$$

$$\mathbf{l}'_{srodq} = \frac{3}{2}n_n^2P_{msr}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{r}'_{sodq} = n_n^2 r'_{s1}\mathbf{I}$$

$$\mathbf{r}'_{rodq} = n_n^2 r'_{r1}\mathbf{I}$$

$$\mathbf{r}''_{sodq} = \begin{bmatrix} r''_{s11} & r''_{s12} & r''_{s13} \\ r''_{s12} & r''_{s22} & r''_{s23} \\ r''_{s13} & r''_{s23} & r''_{s33} \end{bmatrix}$$

$$\mathbf{r}''_{rodq} = \begin{bmatrix} r''_{r11} & r''_{r12} & r''_{r13} \\ r''_{r12} & r''_{r22} & r''_{r23} \\ r''_{r13} & r''_{r23} & r''_{r33} \end{bmatrix}$$

The $\mathbf{I}$ matrix is an identity matrix of appropriate order and the elements of the stator resistance matrices are given by:

$$r''_{s11} = n_n^2(r''_{s2} + r''_{s3})/3$$

$$r''_{s12} = -\frac{3}{\sqrt{2}}[r''_{s2}\cos(\delta+\pi/3) + r''_{s3}\sin(\delta+\pi/6)]$$

$$r''_{s13} = \frac{3}{\sqrt{2}}[r''_{s2}\sin(\delta+\pi/3) - r''_{s3}\cos(\delta+\pi/6)]$$

$$r''_{s22} = \frac{2}{3}[r''_{s2}\cos^2(\delta+\pi/3) + r''_{s3}\sin^2(\delta+\pi/6)]$$

$$r''_{s23} = \frac{1}{3}[-r''_{s2}\sin(2\delta+2\pi/3) + r''_{s3}\sin(2\delta+\pi/3)]$$

$$r''_{s33} = \frac{2}{3}[r''_{s2}\sin^2(\delta+\pi/3) + r''_{s3}\cos^2(\delta+\pi/6)]$$

The rotor resistance matrix is obtained similarly only by replacing the subscript s by r and δ by $\delta - \theta_r$ in the above stator matrices.

Notice that except for the disturbance terms ε^a_s and ε^a_r that have time-varying parameters, all the other parameters of the model given by (16)-(19) are time-invariant. The dependency with ω_r is the same as of a standard odq balanced machine.

The values of $r''_{s12} - r''_{s33}$ and $r''_{r12} - r''_{r33}$ can be approximated by their average values. Besides, if the rotor reference frame is used, only $\boldsymbol{\varepsilon}^r_s$ will depend on θ_r but it may be neglected, except at very low speed.

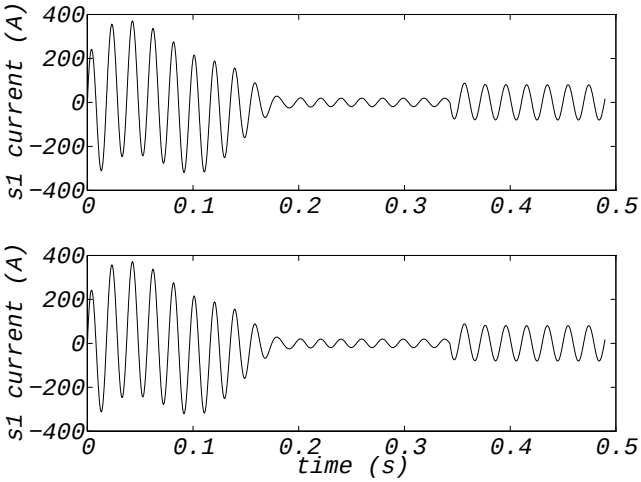
IV. SIMULATION RESULTS

Fig. 1 shows the simulation waveforms of the start-up of a three-phase machine that operates balanced up to $t = 3.5s$. After $t = 3.5s$ the number of coils turns of phase s_1 is changed by 10%. The parameter of the three-phase machine used this simulation study is given in [6]. Fig. 1a presents a comparison of the model proposed (top) and the primitive three-phase model (1)-(5). Fig. 1b presents the dq current in the synchronous reference frame as obtained with the proposed model.

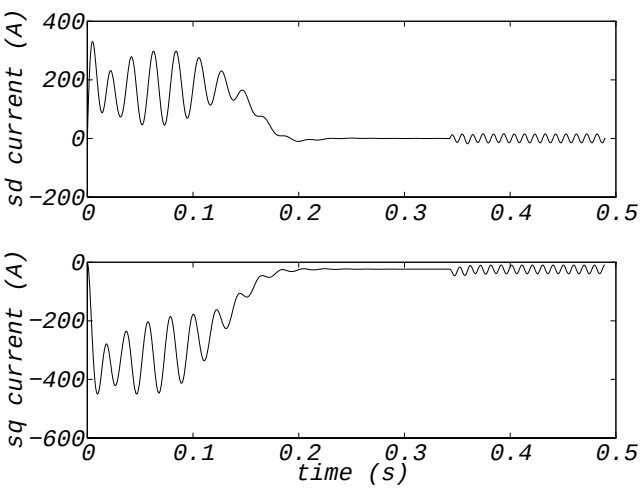
V. CONCLUSIONS

In this paper was presented a mathematical model suitable for the dynamic simulation of an induction machine with interturn short circuit in the stator or rotor windings. With the results from simulation is possible to verify the behaviour of the currents when the short circuit occur.

For the next step, some experiments will take place and the results that will be obtained must validate the model presented.



(a)



(b)

Figura 1 Current waveform during start-up and after changing the number of coil turns of phase s_1 of a three-phase machine. (a) Simulated with the proposed model (top), simulated with the primitive three-phase model. (b) dq currents (bottom) obtained with the proposed model in the synchronous reference frame.

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