Instability mechanisms in indirect field-oriented control of induction motors: mathematical analysis

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Abstract - The influence of the rotor time constant mismatch on the stability of induction motors under indirect field oriented control is analyzed. Different mechanisms for the loss of stability are detected by means of bifurcation analysis. Robustness margins and guidelines for drive commissioning are derived from these results.

1 Introduction

Indirect Field Oriented Control (IFOC) is a well established and widely applied control technique when dealing with high performance induction motor drives [8, 6, 3]. Yet, it was not until very recently that this control scheme has been provided a firm theoretical foundation [10, 9, 4].

The commissioning of an IFOC requires the knowledge of a single motor parameter, namely the rotor time constant, which can vary widely in practice [5, 7]. It has been previously shown that the speed control of induction motors through IFOC is globally asymptotically stable for any constant load torque if the rotor time constant is perfectly known or the error in its estimation is sufficiently small [4, 1].

In this paper we study the loss of performance and stability of IFOC drives due to mismatches in the rotor time constant estimate. Possible instability due to such mismatches is classified into two types in our study: monotonic and oscillatory. We have shown in [1] that monotonic instability occurs for certain values of the mismatch in this estimation and certain load conditions and is associated to the occurrence of saddle-node bifurcations. We have also shown that oscillatory instability is associated with Hopf bifurcations and can also occur for practical values in this mismatch [2].

In this paper we further develop the instability mechanisms in IFOC drives due to mismatches in the rotor time constant estimate. In Section 2 the system modeling and the control equations are given. A convenient parameterization for the system model is introduced. In Section 3, the occurrence of monotonic instability is studied; it is shown that this phenomenon does not depend on the settings of the speed controller. In Section 4, oscillatory instability is considered; design rules for the PI tuning to avoid this kind of instability are derived. Simulation results are provided for a case study to illustrate and clarify the concepts presented.

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2 Problem statement

2.1 Induction motor model

The dynamic model of a current-fed induction motor expressed in a reference frame rotating at synchronous speed can be written in the form

$$\dot{\lambda}_q = -c_1\lambda_q - w_{sl}\lambda_d + c_2 i_q \quad (1)$$

$$\dot{\lambda}_d = -c_1\lambda_d + w_{sl}\lambda_q + c_2 i_d \quad (2)$$

$$\dot{w} = -c_3 w + c_4[c_5(\lambda_d i_q - \lambda_q i_d) - T_m] \quad (3)$$

where λ_q and λ_d are the quadrature axis component and the direct axis component of the rotor flux, respectively, w is the rotor speed, w_{sl}, i_d and i_q stand for the inputs - the slip frequency, the direct axis stator current component and the quadrature axis stator current component respectively; T_m is the load torque, which is assumed constant; the "c" parameters are defined in the appendix and are all positive. In particular, c_1 represents the inverse of the rotor time constant. See [8, 6, 3] for further detail regarding induction motor modeling.

2.2 Indirect field oriented control

In speed regulation applications the IFOC is usually applied along with a PI speed loop. This control strategy is described by the following equations [8, 4]:

$$w_{sl} = \hat{c}_1 \frac{i_q}{i_d} \quad (4)$$

$$i_d = i_d^0 \quad (5)$$

$$i_q = k_p(w_{ref} - w) + k_i \int_0^t (w_{ref}(\zeta) - w(\zeta))d\zeta \quad (6)$$

where $\hat{c}_1$ is an estimate for the inverse rotor time constant c_1, k_p and k_i are the gains of the PI speed controller, w_{ref}

is the constant reference velocity and i_d^0 is some constant which defines the flux level.

The rotor time constant is a critical parameter for indirect field oriented control. If $\hat{c}_1 = c_1$, that is, if we have a perfect estimate of the rotor time constant, we say that the control is tuned, otherwise it is said to be detuned. Accordingly, we define

$$\kappa \triangleq \frac{\hat{c}_1}{c_1} \tag{7}$$

as the degree of tuning. It is clear that $\kappa > 0$ and the control is tuned if and only if $\kappa = 1$.

The closed-loop system (1)-(3) with the control (4)-(6) is a fourth-order system that can be described as:

$$\dot{\lambda}_q = -c_1\lambda_q + c_2 i_q - \frac{\kappa c_1}{i_d^0}\lambda_d i_q \tag{8}$$

$$\dot{\lambda}_d = -c_1\lambda_d + c_2 i_d^0 + \frac{\kappa c_1}{i_d^0}\lambda_q i_q \tag{9}$$

$$\dot{\Delta w} = -c_3\Delta w - c_4[c_5(\lambda_d i_q - i_d^0\lambda_q) - T_m - \frac{c_3}{c_4}w_{ref}] \tag{10}$$

$$\dot{i}_q = k_c\Delta w - k_p c_4[c_5(\lambda_d i_q - i_d^0\lambda_q) - T_m - \frac{c_3}{c_4}w_{ref}] \tag{11}$$

where we have defined $\Delta w \triangleq w_{ref} - w$ and $k_c \triangleq k_i - k_p c_3$. We shall analyze the stability properties of the closed-loop system (8)-(11) and their dependence on the loading conditions, the degree of tuning κ and the settings of the PI controller.

2.3 The tuned system

IFOC drives have to go through an initializing process in order to establish constant magnetic flux inside the motor before they can be turned on. This initialization is known as fluxification phase and consists in applying constant direct axis current (i_d^0) with zero load and zero reference speed until the flux inside the motor reaches a steady-state value. It is easily seen that this steady-state value is $\lambda_d(\infty) = \frac{c_2}{c_1}i_d^0$; meanwhile, the other variables are identically zero during this phase.

After the fluxification phase, the drive is turned on, with the application of load and reference speed, that is, T_m and w_{ref} are allowed to be different from zero. If the system is tuned ($\kappa = 1$), then the terms dependent of λ_q in (8)(9) vanish and it can be shown that the fluxes do not change [8]. Substituting the constant values of the fluxes into the remaining equations (10)-(11) we have

$$\begin{bmatrix} \dot{\Delta w} \\ \dot{i}_q \end{bmatrix} = A \begin{bmatrix} \Delta w \\ i_q \end{bmatrix} + B \begin{bmatrix} w_{ref} \\ T_m \end{bmatrix} \tag{12}$$

$$A = \begin{bmatrix} -c_3 & -\frac{c_4 c_5 c_2 i_d^0}{c_1} \\ (k_i - k_p c_3) & -\frac{k_p c_4 c_5 c_2 i_d^0}{c_1} \end{bmatrix}$$

$$B = \begin{bmatrix} c_3 & c_4 \\ k_p c_3 & k_p c_4 \end{bmatrix}$$

which is a second-order *linear* system. We shall refer to the dynamic system (12) as the *tuned system*. A block-diagram of the tuned system is given in Figure 1,

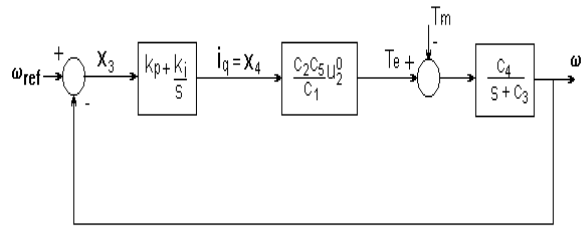

Figure 1: The tuned system.

where the plant is the induction motor under perfect field-orientation (tuned).

As the design of the PI controller is in general based on the tuned system, our concern in this paper is to provide guidelines for this design such that stability is guaranteed for a practical range of κ.

3 Monotonic instability

Let us define the dimensionless variables $r \triangleq \frac{i_q^e}{i_d^0}$ and $r^* \triangleq \frac{T_e^0 c_1}{c_5 c_2 (i_d^0)^2}$, where $T_e^0 \triangleq T_m + \frac{c_3}{c_4}w_{ref}$ is the total demanded torque. The constant r^* represents the system loading, since it is proportional to the electrical torque developed in steady-state. It also coincides with the steady-state ratio between the q-axis and the d-axis components of stator currents in the tuned condition, so that it coincides with r in that operating condition.

The equilibrium values of the fluxes can be obtained from (8)(9) as

$$\begin{bmatrix} c_1 & c_1\kappa r \\ -c_1\kappa r & c_1 \end{bmatrix} \begin{bmatrix} \lambda_q^e \\ \lambda_d^e \end{bmatrix} = \begin{bmatrix} \frac{1}{c_2 i_d^0} r \\ c_2 i_d^0 \end{bmatrix} \tag{13}$$

Solving (13) for λ_q^e, λ_d^e yields

$$\lambda_q^e = \frac{c_2}{c_1} i_d^0 \frac{1-\kappa}{1+\kappa^2 r^2} r \tag{14}$$

$$\lambda_d^e = \frac{c_2}{c_1} i_d^0 \frac{1+\kappa r^2}{1+\kappa^2 r^2} \tag{15}$$

Now, from (10) and (11) it is clear that

$$\Delta\omega^e = 0 \tag{16}$$

$$c_5(\lambda_d^e r - \lambda_q^e) i_d^0 = T_e \tag{17}$$

Substituting (14)(15) into (17) we get

$$\frac{c_2}{c_1}(i_d^0)^2 \frac{\kappa r^3 + \kappa r}{1+\kappa^2 r^2} = \frac{T_e}{c_5} \tag{18}$$

Then, from (18), it follows that r must satisfy the following polynomial equation

$$\kappa r^3 - r^*\kappa^2 r^2 + \kappa r - r^* = 0 \tag{19}$$

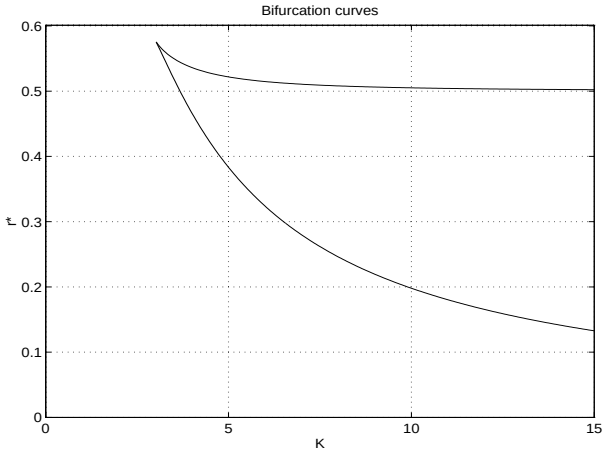

Figure 2: Saddle-node bifurcation diagram.

Collecting equations (14), (15) and (16) and the definition of r we can write the equilibrium point as

$$\begin{bmatrix} \lambda_q^e \\ \lambda_d^e \\ \Delta\omega^e \\ i_q^e \end{bmatrix} = \begin{bmatrix} \frac{c_2 i_d^0}{c_1} \frac{1-\kappa}{1+\kappa^2 r^2} r \\ \frac{c_2 i_d^0}{c_1} \frac{1+\kappa r^2}{1+\kappa^2 r^2} \\ 0 \\ i_d^0 \; r \end{bmatrix} \tag{20}$$

The equilibrium is parameterized in terms of a single dimensionless quantity r, which satisfies equation (19). This is a third order polynomial equation whose coefficients are also dimensionless and depend only on the degree of tuning κ and the motor load as denoted by r^*.

The real solutions of equation (19) give the equilibrium values of r for any given degree of tuning - κ - and any given load - r^*. It is clear that equation (19) has at least one and at most three real solutions, depending on the particular values of κ and r^*.

The complete characterization of equilibria is given in Figure 2. The system has a unique equilibrium outside the region delimited by the two curves and three equilibria inside it; exactly on the curves, two equilibria are present [1]. Notice that this characteristic is not dependent on any motor or control parameters. Monotonic instability is not observed in any case for $\kappa < 3$.

As the parameters vary through one of these bifurcation curves monotonic instability can be observed. One such case is illustrated in Figure 3, which shows the response of an IFOC drive with $\kappa = 4$ to a slow increase in load, starting from zero. As the load crosses the value given by the upper curve in figure 2, the motor escapes from the operating point to a much higher current level, which would most likely be unbearable in practice.

Carefully designed IFOC drives usually have no more than 100% error in the estimate of the rotor time constant. Hence, practical values of κ are usually below 2 and monotonic instability is not expected to occur. Yet, the above result has important operational and design implications. First, it shows the importance - not always recognized - of a careful estimate of the rotor time constant: if the estimate is poor to start with then the temperature variations during operation can cause κ to go much above 3. Second, proximity to instability usually implies poor performance, so that unacceptable performance can be observed for practical mismatches even for a carefully designed IFOC, that is, even for $\kappa < 2$.

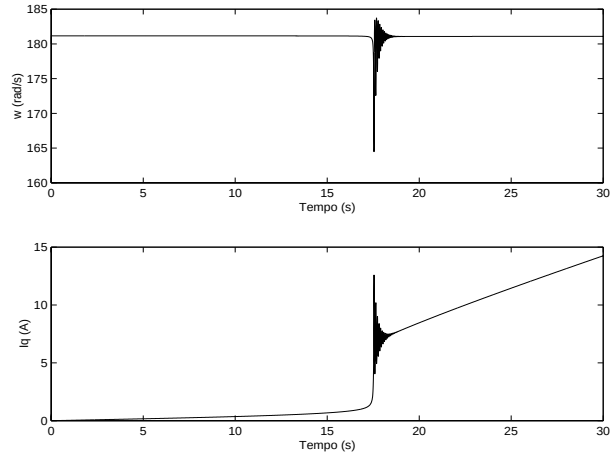

Figure 3: Simulation for slow increase in load with rotor time constant mismatch $\kappa = 4$.

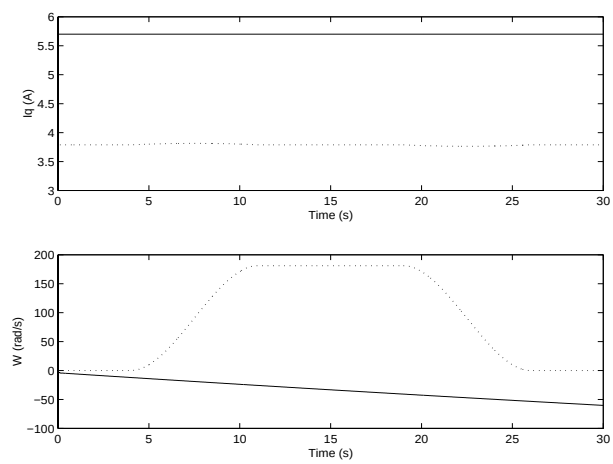

Figure 4: Simulation of load step with current limits enforced for $\kappa = 1$ (dotted line) and $\kappa = 2$ (solid line).

Indeed, Figure 4 shows the simulation of an IFOC drive in which current limits are enforced. In this case, the reduction in the delivered torque due to the rotor time constant mismatch causes the motor to be unable to drive the load, since the supply current is limited. The performance loss is clear by comparing with the response in the tuned case.

4 Oscillatory instability

Oscillatory instability is a major concern, since it can occur for values of κ very close to unity. Also, this occurrence is dependent on the settings of the PI speed controller. Let us study this kind of instability. From (12), the closed-loop eigenvalues of the tuned system are

the roots of the characteristic polynomial

$$\begin{aligned} p_T(s) &= (s+c_3)(s+k_pK)+(k_i-k_pc_3)K \\ &= s^2+(c_3+k_pK)s+k_iK \end{aligned} \tag{21}$$

where $K \triangleq \frac{c_2c_4c_5i_d^0}{c_1}$. Then the PI parameters k_p and k_i can be chosen to arbitrarily assign the closed-loop eigenvalues of the tuned system. Indeed, let the desired closed-loop characteristic (Hurwitz) polynomial be written as

$$s^2+a_1s+a_0, \qquad a_0>0,\ a_1>0 \tag{22}$$

Then, equating the coefficients in (21) and (22) yields

$$k_p = \frac{a_1-c_3}{K} \tag{23}$$

$$k_i = \frac{a_0}{K} \tag{24}$$

Once the closed-loop poles are chosen the parameters k_p and k_i can be calculated from (23)(24).

Calculating the Jacobian matrix of the complete (not tuned) closed-loop system (8)-(11) and substituting (23)(24), the Jacobian matrix can be written in terms of a_1, a_0 as

$$J = \begin{bmatrix} -c_1 & -c_1\kappa r & 0 & c_2(1-\kappa)/(1+\kappa^2r^2) \\ c_1\kappa r & -c_1 & 0 & c_2\kappa(1-\kappa)/(1+\kappa^2r^2)r \\ \frac{c_1}{c_2}K & -\frac{c_1}{c_2}Kr & -c_3 & -K(1+\kappa r^2)/(1+\kappa^2r^2) \\ \frac{c_1}{c_2}(a_1-c_3) & -\frac{c_1}{c_2}(a_1-c_3)r & \frac{a_0-c_3(a_1-c_3)}{K} & -(a_1-c_3)(1+\kappa r^2)/(1+\kappa^2r^2) \end{bmatrix} \tag{25}$$

The roots of the characteristic polynomial of the Jacobian matrix J are the eigenvalues of this matrix and hence determine the stability of the system (8)-(11). This polynomial is given by

$$P(s) = s^4+p_3s^3+p_2s^2+p_1s+p_0 \tag{26}$$

whose coefficients can be written in the form

$$p_3 = (a_1-c_3)v_2+c_3+2c_1 \tag{27}$$

$$p_2 = a_0v_2+c_1\left(2c_3+(a_1-c_3)v_1+c_1(1+\kappa^2r^2)\right) \tag{28}$$

$$p_1 = c_1a_0v_1+c_1^2\left(c_3(1+\kappa^2r^2)+(a_1-c_3)\kappa v_0\right) \tag{29}$$

$$p_0 = c_1^2a_0\kappa v_0 \tag{30}$$

where we have defined

$$v_2 \triangleq \frac{1+\kappa r^2}{1+\kappa^2r^2} \tag{31}$$

$$v_1 \triangleq \frac{\kappa(3-\kappa)r^2+\kappa+1}{1+\kappa^2r^2} \tag{32}$$

$$v_0 \triangleq \frac{\kappa^2r^4+(3-\kappa^2)r^2+1}{1+\kappa^2r^2} \tag{33}$$

We verify that its coefficients - and therefore its roots - do not depend explicitly on the flux level. Hence, the local stability properties of the closed-loop system depend, for a given motor, only on κ, r^* and the dynamics chosen for the tuned system (expressed by a_1 and a_0). Moreover, the only motor physical parameter which appear in the characteristic polynomial are friction (c_3) and the rotor time constant (or its inverse, c_1). For a given motor, conditions for the stability out of the tuned condition depend only on three parameters:

- the drive loading;
- the degree of tuning;
- the dynamics assigned for the tuned system.

We have thus reduced the analysis of a general IFOC drive to three parameters. The stability out of the tuned condition has been parameterized in terms of the dynamics assigned for the tuned condition. Then we can look for design rules which will guarantee robustness margins by looking just at the tuned system. This condition, in its turn, can be made a function of the rotor time constant, which provides a quite generic design rule for the PI speed controller.

In most cases temperature variations inside the rotor can cause the rotor time constant to vary more than 50% but not more than 100% [5]. Hence, $\kappa < 2$ in most practical cases. Although stability is only lost through the occurrence of a bifurcation, a nearby bifurcation is enough to cause unsatisfactory transient behavior. Indeed, if the system is allowed to present a bifurcation for, say, $\kappa = 2.1$, very poor transient performance will be experimented for $\kappa \approx 2$, as shown by an example given below.

On the other hand, for $\kappa > 3$ there always exist a range of r^* for which an unstable equilibrium point exists, regardless of the PI settings. Taking these considerations into account, we choose to search for PI settings that will avoid Hopf bifurcations in the range 0 to 3 for any loading condition.

Theorem 1 *Let s_1, s_2 denote the eigenvalues of the tuned system (12) with $c_3 = 0$. If $s_1 = s_2 = -18.c_1$ then the equilibrium of the system (8)-(11) is locally asymptotically stable for all $(\kappa,\ r^*) \in \mathcal{D} \triangleq \{(0,3] \times \Re\}$.*

◇

Proof: *First recall that the equilibrium is unique inside $\mathcal{D}$. The Jacobian matrix of (8)-(11) around the equilibrium is given by (25) and its characteristic polynomial by (26). If the eigenvalues are both equal to $-18c_1$ then $a_1 = 2.18.c_1$ and $a_0 = 18^2.c_1^2$. Substitution of a_1, a_0 into (25) and calculation of the characteristic polynomial yields*

$$P(s) = s^4+\alpha_3s^3+\alpha_2s^2+\alpha_1s+\alpha_0 \tag{34}$$

where

$$\alpha_3 = \frac{2c_1[(18\kappa+\kappa^2)r^2+19]}{1+\kappa^2r^2} \tag{35}$$

$$\alpha_2 = \frac{c_1^2[\kappa^4r^4+(432\kappa-34\kappa^2)r^2+36\kappa+361]}{1+\kappa^2r^2} \tag{36}$$

$$\alpha_1 = \frac{36c_1^3[\kappa^3r^4+(30\kappa-\kappa^3-9\kappa^2)r^2+10\kappa+9]}{1+\kappa^2r^2} \tag{37}$$

$$\alpha_0 = \frac{324c_1^4\kappa[\kappa^2r^4+(3-\kappa^2)r^2+1]}{1+\kappa^2r^2} \tag{38}$$

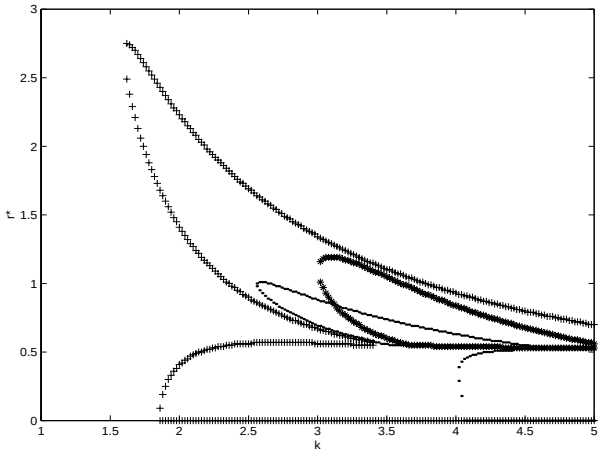

Figure 5: Hopf bifurcation for: (+) $s_{1,2} = (-1 \pm \jmath 10)c_1$, (−) $s_{1,2} = (-5 \pm \jmath 10)c_1$, (∗) $s_1 = s_2 = -23c_1$.

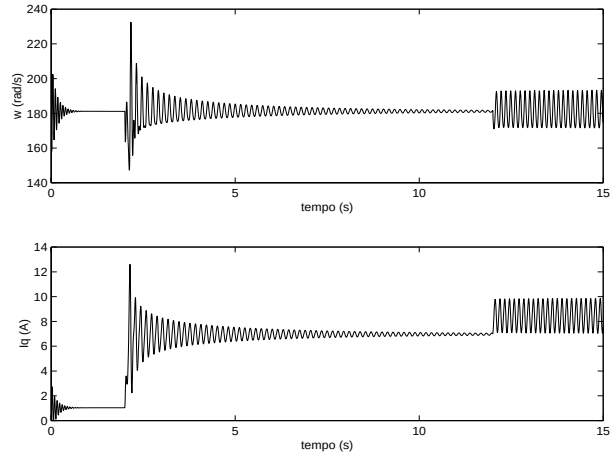

Figure 6: Speed and quadrature axis current for load torque variations; $\kappa = 2.7$, $s_{1,2} = (-1.2 \pm \jmath 7)c_1$.

The Theorem is proved by applying the Routh-Hurwitz criterion to this polynomial[1] *and verifying that all the coefficients of the Routh-Hurwitz array are positive for all values of* $(\kappa,\ r^*) \in \mathcal{D}$.

□

The Theorem states that robust stability of the IFOC out of the tuning condition is guaranteed by proper assignment of the dynamics of the tuned system. These dynamics can be arbitrarily assigned by the settings of the PI speed loop according to (23)(24). Stability robustness against mismatches in the rotor time constant is guaranteed if the tuned system is designed to be non-oscillatory and we are not too greedy regarding its dynamic performance.

Figure 5 shows the bifurcation curves for three different choices of the closed-loop eigenvalues. Each curve represents the locus of the points at which a Hopf bifurcation occurs for each different PI setting. For each setting, the system is locally stable outside the corresponding curve and unstable inside it. Two bifurcation curves are observed: one for light load (around $r^* = 0$) and another one for medium/heavy load. As the damping of the poles is decreased, the lower (light load) branch occurs for lower values of κ, eventually approaching practical values of this parameter. On the other hand, if the poles are moved too far away towards the left, the higher (medium/heavy load) branch tends to occur for lower values of κ.

In order to illustrate the effects of Hopf bifurcations on the system's performance, we present simulations for a $1cv$ induction motor - see parameters in the appendix. Figure 6 shows the system response to several steps of load torque, starting from no load until the system reaches a Hopf bifurcation. The normalized load factor r^* is stepped to 2.7 at $t = 0$ s, then to 4.3 at $t = 2$ s and finally to 4.8 at $t = 12$ s. It is clear that the dynamic performance of the system is deteriorated as it approaches the bifurcation, becoming very poor much before reaching it. After the bifurcation occurs, a sustained oscillation is observed.

[1] This straightforward procedure is omitted here in the interest of brevity.

5 Discussion

The occurrence of instability in indirect field oriented induction motor control as a result of mismatches in the rotor time constant has been analyzed. Two classes of instability have been identified: monotonic and oscillatory. Monotonic instability has been shown to be independent of the speed loop settings, contrary to oscillatory instability.

The parameterization derived in section 3 clarified such dependence by showing that for an actual motor the existence of Hopf bifurcations depends only on the closed-loop poles assigned for the tuned system, the degree of tuning κ and the load condition.

The PI setting is closely related to the appearance of oscillatory instability. Poor PI setting can cause oscillatory instability when the rotor time constant is poorly known basically in two different ways. One is to make the closed-loop response highly oscillatory by assigning complex conjugate eigenvalues with low damping for the tuned system. The second one is to force the closed-loop response to be very fast by choosing the closed-loop eigenvalues too far away to the left in the complex plane. The second one is more likely to occur, since it seems a good choice in terms of performance of the tuned system, contrary to the first one.

Rules for the PI setting were provided in Theorem 1 ensuring robust stability of IFOC for a practical range of κ. The sufficient condition pointed out that the closed loop poles of the tuned system should be chosen neither too far to the left hand side of the complex plane nor with too low damping. Simulation results were presented

showing the performance loss due to rotor time constant mismatch and poor PI setting (closeness to bifurcations).

A List of motor parameters

- c_1: inverse of the rotor time constant;
- $c_2 = L_m c_1$ where L_m is the mutual stator-rotor inductance;
- c_4: inverse of the momentum of inertia;
- $c_3 = Bc_4$ where B is the friction coefficient;
- $c_5 = \frac{3PL_m}{2L_r}$ where L_r is the rotor inductance and P is the number of poles.

B Data for the case study

All the simulations presented are for a real 1 *cv* squirrel-cage induction motor with the following parameters:

c_1	$13.67\ s^{-1}$
c_2	$1.56\ H.s^{-1}$
c_3	$0.59 s^{-1}$
c_4	$1,176 kg^{-1}m^{-2}$
c_5	2.86

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