Small Signal Stability For Single Phase Inverter Connected to Stiff AC System

Ernane Antônio Alves Coelho

Electrical Engineering Department
Federal University of Uberlândia
Av. João Naves de Ávila 2160
38400-902 Uberlândia/MG - Brazil
e-mail: ernane@ufu.br
Tel.: +55 34 236-5099

Porfírio Cabaleiro Cortizo
Pedro Francisco Donoso Garcia

Electrical Engineering Research and Development Center
Federal University of Minas Gerais
Av. Antônio Carlos 6627 - Pampulha
31270-901 Belo Horizonte/MG - Brazil
e-mail: porfirio@novell.cpdee.ufmg.br
Tel.: +55 31 499-5465 - Fax: +55 31 499-5480

Abstract

A control technique based in frequency and voltage droop is used to control a single phase inverter connected to stiff AC system. The small signal analysis of the system is presented, which allows to obtain the behavior of the system in function of frequency and voltage droop. Simulation results and experimental results for the different frequency and voltage droop are presented.

1 Introduction

Dynamic characteristics of a power system can be obtained by small signal analysis using linear techniques [1]. Traditionally, this technique is used in study of stability of a single machine connected to stiff AC system. Stability study of single phase inverter connected to stiff AC system can be made in the same way.

A synchronous machine presents a rotor inertial that limits the speed derivative or frequency variation. Furthermore, there is a natural connection between the operating frequency and the output power, that is, to constant mechanical input power , the operating frequency decreases when the output power increases. Two or more synchronous machines connected in parallel tends to remain in synchronism due to synchronizing torque component. The system stability depends on the existence of sufficient synchronizing torque component and damping torque component. Voltage stability depends on the equilibrium in the demand for reactive power [1].

In the other hand, the power inverters present neither a natural connection between frequency and output power nor connection between output voltage and reactive output power. In order to connect a single phase inverter to stiff AC system, it is necessary to create these connections by control system to remain operation stable.

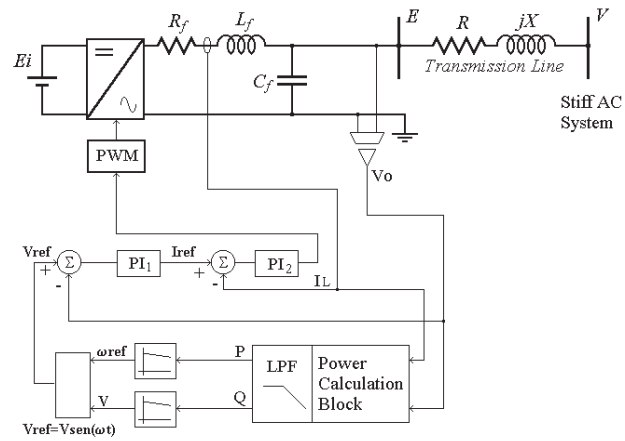

Figure 1: Single phase inverter connected to stiff AC system

2 Control Scheme

The control scheme used to connect a single phase inverter to stiff AC Power is shown in Figure 1. The inverter presents a classical PWM controller with inner current loop and outer voltage loop using PI compensators. The sinusoidal reference for controller is obtained in reference block from the amplitude signal and frequency signal defined by power droop curves. Active power and reactive power are obtained in power calculation block using a computational algorithm proposed in [3].

3 Small Signal Analysis

Inverter output frequency ω and inverter output voltage E will be controlled by the droop characteristics defined by equations 1 and 2, respectively, which are represented in Figure 2.

$$\omega = \omega_o - k_p P \quad (1)$$

$$E = E_o - k_v Q \quad (2)$$

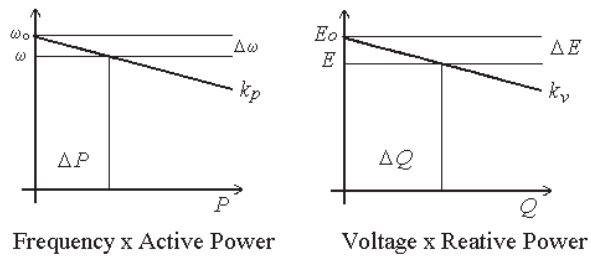

Figure 2: Frequency and voltage droop

The active power and the reactive power transferred from the inverter to stiff AC system is given by equations 3 and 4. In this case, these expressions includes de power losses and reactive power through inductive reactance associated with the transmission line .

$$P = \frac{1}{R^2+X^2}(RE^2 - REVcos\delta + XEVsen\delta) \quad (3)$$

$$Q = \frac{1}{R^2+X^2}(XE^2 - XEVcos\delta - REVsen\delta) \quad (4)$$

Considering small disturbances around the state of equilibrium defined by (δ_e, E_e, V_e), the above equations can be linearized, then we have

$$\Delta\omega = \frac{\partial\omega}{\partial P}\Delta P \quad (5)$$

$$\Delta E = \frac{\partial E}{\partial Q}\Delta Q \quad (6)$$

$$\Delta P = \frac{\partial P}{\partial E}\Delta E + \frac{\partial P}{\partial \delta}\Delta\delta \quad (7)$$

$$\Delta Q = \frac{\partial Q}{\partial E}\Delta E + \frac{\partial Q}{\partial \delta}\Delta\delta \quad (8)$$

where Δ denotes the small deviation of the variable from the equilibrium point. Substituting for P and Q given by equations 3 and 4 and calculating the partial derivatives, we obtain

$$\Delta\omega = -k_p\Delta P \quad (9)$$

$$\Delta E = -k_v\Delta Q \quad (10)$$

$$\Delta P = k_{pe}\Delta E + k_{pd}\Delta\delta \quad (11)$$

$$\Delta Q = k_{qe}\Delta E + k_{qd}\Delta\delta \quad (12)$$

where

$$k_{pe} = \frac{1}{R^2+X^2}(2RE_e - RV_e cos\delta_e + XV_e sen\delta_e) \quad (13)$$

$$k_{pd} = \frac{1}{R^2+X^2}(RE_eV_e sen\delta_e + XE_eV_e cos\delta_e) \quad (14)$$

$$k_{qe} = \frac{1}{R^2+X^2}(2XE_e - XV_e cos\delta_e - RV_e sen\delta_e) \quad (15)$$

$$k_{qd} = \frac{1}{R^2+X^2}(XE_eV_e sen\delta_e - RE_eV_e cos\delta_e) \quad (16)$$

In order to ensure the control laws defined by equations 1 and 2, it is necessary to measure output power of the single phase inverter. This measuring block uses a low pass filter which pass band is much smaller than the pass band of the voltage control of inverter, then the performance of control will have a hard influence of this filter. Furthermore, the pass band of the voltage control can be increased using several techniques, as can be seen in [2]. Therefore, we can consider the inverter like an ideal voltage source, where we can control the amplitude and frequency. Then, the active and reactive output power obtained by measuring block can be given by equations 17 and 18, where ω_f is the cut-off frequency of the measuring filter.

$$\Delta P_{mes}(s) = \frac{\omega_f}{s+\omega_f}\Delta P(s) \quad (17)$$

$$\Delta Q_{mes}(s) = \frac{\omega_f}{s+\omega_f}\Delta Q(s) \quad (18)$$

Therefore, it follows from above equations

$$\Delta\omega(s) = -\frac{k_p\omega_f}{s+\omega_f}(k_{pe}\Delta E(s) + k_{pd}\Delta\delta(s)) \quad (19)$$

$$\Delta E(s) = -\frac{k_v\omega_f}{s+\omega_f}(k_{qe}\Delta E(s) + k_{qd}\Delta\delta(s)) \quad (20)$$

Rearranging, we have

$$\Delta\omega(s) = -\frac{k_p\omega_f}{s+\omega_f}\left(k_{pd}\Delta\delta(s) - \frac{k_vk_{pe}k_{qd}\omega_f}{s+\omega_f+k_vk_{qe}\omega_f}\Delta\delta(s)\right) \quad (21)$$

Since

$$\Delta\omega(s) = s\Delta\delta(s) \quad (22)$$

It implies that

$$s^3\Delta\delta(s) + as^2\Delta\delta(s) + bs\Delta\delta(s) + c\Delta\delta(s) = 0 \quad (23)$$

where

$$a = (2 + k_vk_{qe})\omega_f \quad (24)$$

$$b = (k_pk_{pd} + k_vk_{qe}\omega_f + \omega_f)\omega_f \quad (25)$$

$$c = (k_{pd} + k_vk_{pd}k_{qe} - k_vk_{pe}k_{qd})k_p\omega_f^2 \quad (26)$$

The homogeneous equation 23 describes the free motion of the system for small disturbances around equilibrium point (δ_e, E_e, V_e). Then, the system response can be analyzed by the characteristic equation 27

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0 \quad (27)$$

4 Simulation Results

Simulation results for a single phase inverter connected to stiff AC system are shown in order to validate the equation 23. Two examples for different frequency and voltage droop coefficients are presented in simulation studies.

4.1 Example I

Taking the system showed in Figure 1 with the parameters presented in table 1, tuning the offset of frequency droop in way that inverter provides 1005 W at stiff AC system frequency and tuning the offset of voltage droop in way that inverter provides 525 VAR at output voltage of 223 V, we can obtain the behavior of angle $\Delta\delta$ of the system.

Variable	Value	Unit
Line impedance	$0.2+j1$	Ω
Cut-off frequency of measuring filter(ω_f)	37.7	rd/s
Frequency voltage droop coefficient (k_p)	0.0001	rd/s/W
Frequency voltage droop coefficient (k_v)	0.0001	V/VAR
Apparent power in stiff AC system	$1000+j500$	VA
Inverter output apparent power	$1005+j525$	VA
Stiff AC system voltage (V)	220	V (rms)
Inverter output voltage (E)	223	V (rms)
Stiff AC system frequency (ω)	377	rd/s
Inverter-stiff AC system lead angle($\Delta\delta$)	0.0183	rd

Table 1: System parameters and equilibrium point

Considering that the initial active and reactive output power are nulls and solving the characteristic equation 27, we obtain

$$\lambda_1 = -42.03 \quad (28)$$
$$\lambda_2 = -28.41 \quad (29)$$
$$\lambda_3 = -5.77 \quad (30)$$

The system presents three negative real poles and consequently it has a stable damped response defined by equation 32. Figure 3 presents the phase response of single phase inverter connected to stiff AC system obtained by equation 32 and simulation in Pspice. We can observe that the system is well represented by equation 32.

$$\Delta\delta = 0.0008\, e^{-42.03\, t} - 0.0011\, e^{-28.41\, t} - 0.018\, e^{-5.77\, t} \quad (31)$$
$$\delta = \delta_e + \Delta\delta \quad (32)$$

The active and reactive power are presented in Figure 4. The oscillations at 120 Hz are not completely eliminated by the measuring filter from the power measurements. The reactive power flux is faster than active power flux and both presents a damped response.

Figure 5 presents the output voltage and current supplied by inverter at starting of simulation.

4.2 Example II

Using the same conditions and method for Example I, but increasing k_p and k_v in 10 times, we have

$$\lambda_1 = -52.26 \quad (33)$$
$$\lambda_2 = -15.66 + j36.52 \quad (34)$$
$$\lambda_3 = -15.66 - j36.52 \quad (35)$$

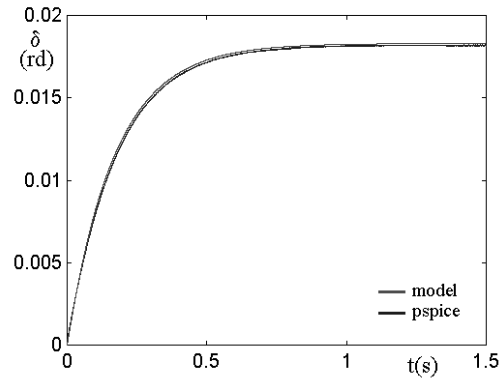

Figure 3: Phase response of inverter

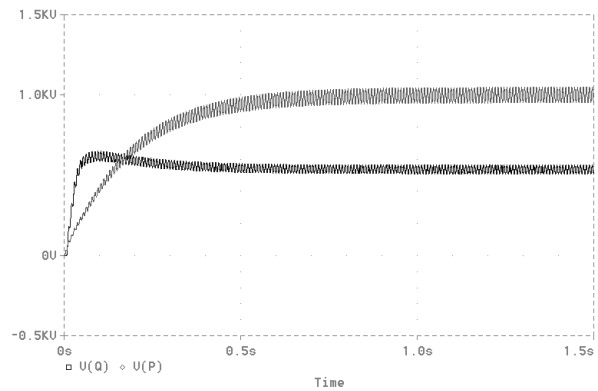

Figure 4: Active and reactive power

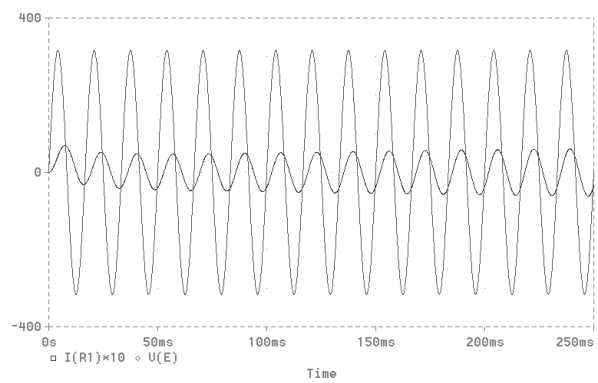

Figure 5: Output voltage and output current

Now the system presents a negative real pole and complex conjugated poles, therefore it presents a oscillatory response. The negative real part of poles implies that oscillations are damped. Figure 6 presents the phase response of single phase inverter connected to stiff AC system obtained calculations with above poles and simulation in Pspice. The results are very close again.

The active and reactive power are presented in Figure 7. The power flux in this case is faster than Example I, but it is oscillatory.

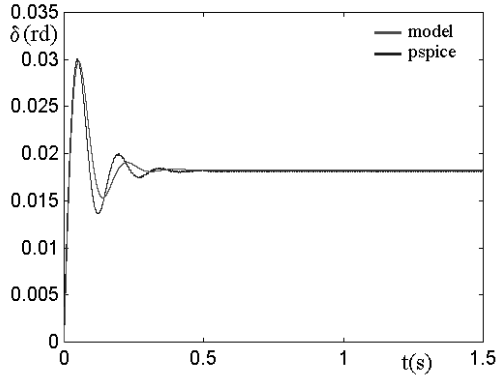

Figure 6: Phase response of inverter

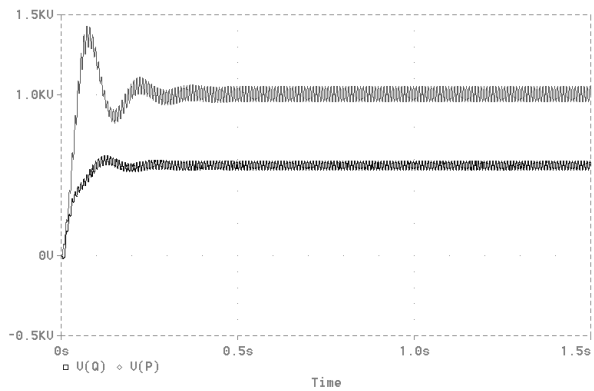

Figure 7: Active and reactive power

Figure 8 presents the output voltage and current supplied by inverter at starting of simulation. The oscillatory effect can be observed in current waveform.

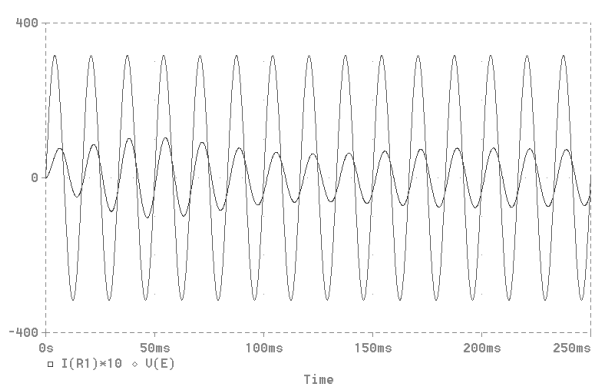

Figure 8: Output voltage and output current

5 Experimental Results

A prototype was assembled in order to validate theoretical studies. Its scheme is shown in figure 9. It consists in a single phase IGBT inverter connected to stiff AC system by inductor. The inverter presents a classical PI control with inner current loop and outer voltage loop. The DC bus was obtained by a three phase rectifier, which implies that only positive active power flux is possible. The part of scheme shown in shade area consists in the power flux control, which uses a 133MHz PC computer and a acquisition system (PC30DS). Besides power flux control function, the control sotfware presents a PLL block used to start the system.

First of all, IGBT gate drivers are disabled and then the switch SW1 is closed. An AC voltage appears in the filter capacitor of inverter due to stiff AC system. Then the reference voltage is synchronized with output voltage of inverter (capacitor voltage) by the PLL block. When reference voltage and output voltage are in phase, SW2 (software switch) is changed from state 1 to state 2, and IGBT gate drivers are simultaneously enabled through parallel port.

In order to reduce oscillations at 120Hz in power signal, a second order low-pass filter was used during the experimental tests, which respective transfer function is described by equation 36.

$$filter\ transfer\ function = \frac{\omega_f^2}{s^2 + 2\xi\omega_f s + \omega_f^2} \tag{36}$$

Then, using the same procedure shown in section 3, we can obtain the equation 37, which describes the behavior of the system around the equilibrium point.

$$\begin{aligned} s^5\Delta\delta(s) \quad &+ \quad as^4\Delta\delta(s) + bs^3\Delta\delta(s) + \\ cs^2\Delta\delta(s) \quad &+ \quad ds\Delta\delta(s) + e\Delta\delta(s) = 0 \end{aligned} \tag{37}$$

where

$$a = 4\xi\omega_f \tag{38}$$
$$b = \omega_f^2(2 + k_v k_{qe} + 4\xi^2) \tag{39}$$
$$c = \omega_f^2(k_p k_{pd} + 4\xi\omega_f + 2\xi k_v k_{qe}\omega_f) \tag{40}$$
$$d = \omega_f^3(2\xi k_p k_{pd} + \omega_f + k_v k_{qe}\omega_f) \tag{41}$$
$$e = k_p\omega_f^4(k_{pd} - k_v k_{pe} k_{qd} + k_v k_{pd} k_{qe}) \tag{42}$$

Four tests for different frequency and voltage droop and different power factor were done and the respective results are presented next. In all tests, the power flux control and IGBT gate drivers were enabled 0.05 seconds after acquisition system has started to save data. In each test, the frequency droop and voltage droop offsets were tuned in way that inverter provides the specified active and reactive power flux, respectively.

The acquisition system presents a digital second order filter with cut-off frequency at 1kHz in order to eliminate the inverter switching noise. The samples are transferred to computer using DMA techniques. Both acquisition system and digital power control loop runs at 5kHz.

Inverter frequency is presented instead of inverter-stiff AC system phase due to difficulty to measure the phase of

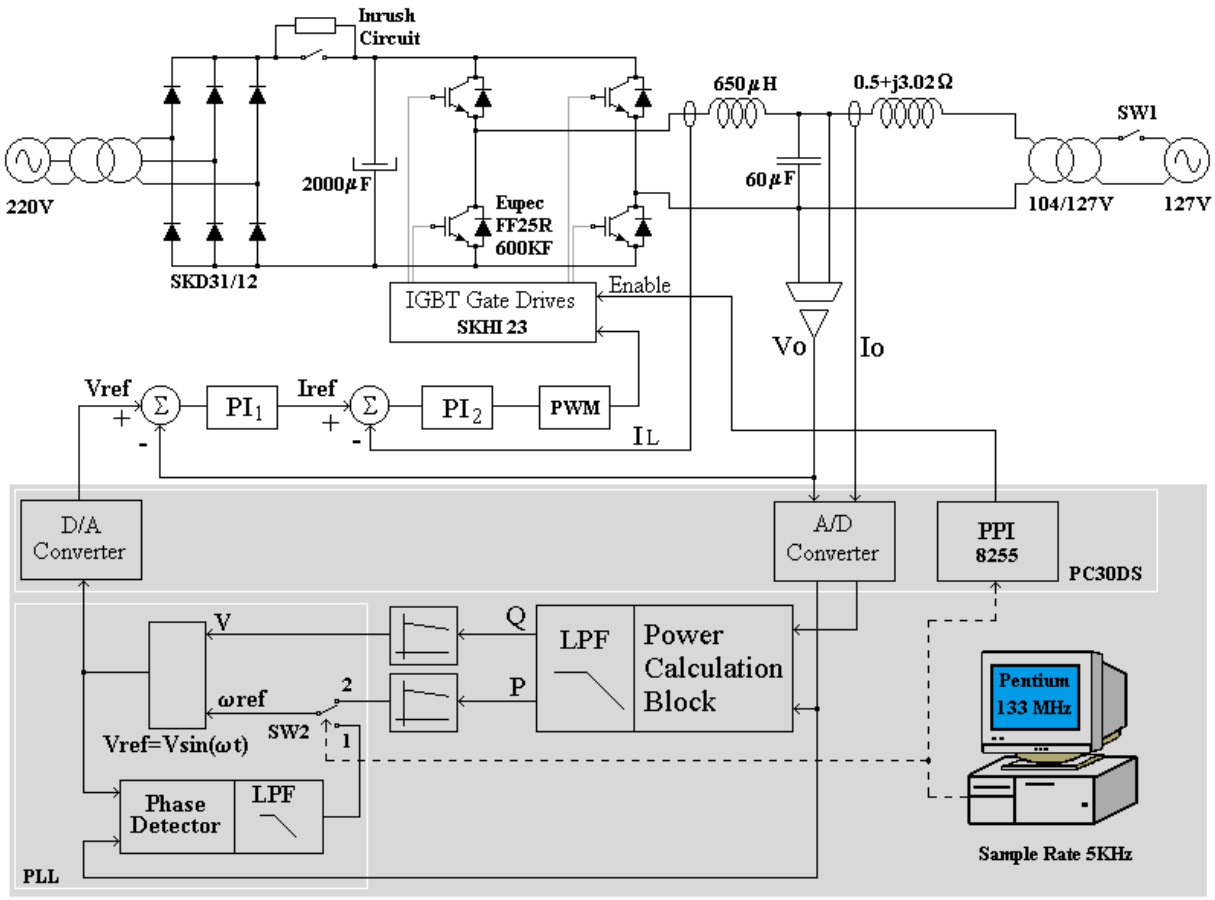

Figure 9: Prototype

the signals, which is compared with the frequency defined by equations 22 and 37(small signal model). Before the power control and IGBT gate drivers are enabled, the frequency shown in figures is defined by PLL block.

Variable	Value	Unit
Line impedance	$0.5+j3.02$	Ω
Cut-off frequency of measuring filter(ω_f)	75.4	rd/s
filter damping factor	0.7	
Frequency voltage droop coefficient (k_p)	0.005	rd/s/W
Frequency voltage droop coefficient (k_v)	0.005	V/VAR
Apparent power in stiff AC system	$500+j250$	VA
Inverter output apparent power	$514+j337$	VA
Stiff AC system voltage (V)	104	V (rms)
Inverter output voltage (E)	114	V (rms)
Stiff AC system frequency (ω)	377	rd/s
Inverter-stiff AC system lead angle($\Delta\delta$)	0.1165	rd

Table 2: Parameters for Test I

The parameters used in test I are presented in table 2. Figure 10 shows the active and reactive power provided by inverter. Before the power control and IGBT gate drivers are enabled, inverter filter capacitor provides reactive power to the stiff AC system, then the reactive power doesn't start from zero. Both power fluxes present

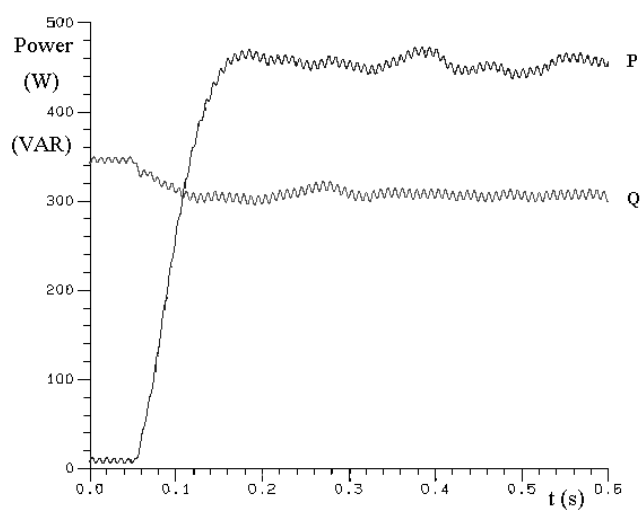

Figure 10: Test I - Active and Reactive Power

a damping response. Figure 11 presents the experimental inverter frequency and the inverter frequency defined by the model. It can be seen that the real system has a higher damping than the model. Figure 12 shows the

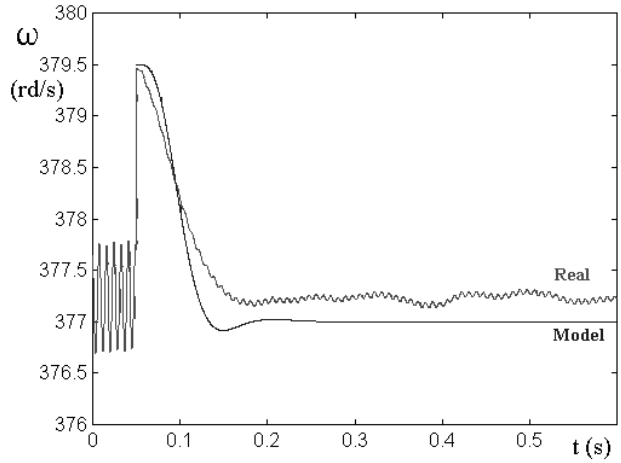

Figure 11: Test I - Real and Model Inverter Frequency

inverter output voltage and current. The current scale is 10 times higher than normal scale.

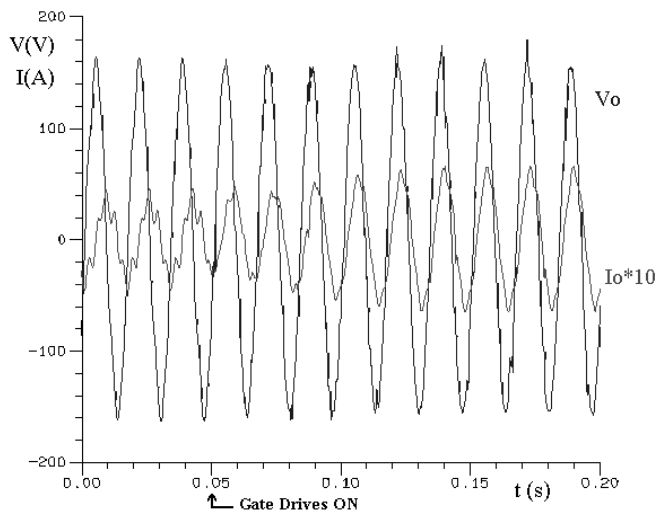

Figure 12: Test I - Inverter Ouput Voltage and Current

The parameters used in test II are presented in table 3. Now, the reactive power flux is negative. The respective results are presented in figures 13, 14 and 15. The reactive power flux is faster than active power flux, then it causes a fast inverter output voltage reduction, which implies in a transient negative active power flux. It can be seen in figure 13 and its consequence can be observed in frequency (figure 14). Figure 15 shows that the inverter output current leads the inverter ouput voltage.

Table 4 shows the parameters used in test III. The coefficients k_p and k_v were increased twice. In this case, both active and reactive power fluxes have oscillatory response, as can be seen in figure 16. The oscillations can be observed in the inverter frequency (figure 17) and in the output current (figure 18).

Variable	Value	Unit
Line impedance	$0.5+j3.02$	Ω
Cut-off frequency of measuring filter(ω_f)	75.4	rd/s
filter damping factor	0.7	
Frequency voltage droop coefficient (k_p)	0.005	rd/s/W
Frequency voltage droop coefficient (k_v)	0.005	V/VAR
Apparent power in stiff AC system	$500-j250$	VA
Inverter output apparent power	$514-j163$	VA
Stiff AC system voltage (V)	104	V (rms)
Inverter output voltage (E)	100	V (rms)
Stiff AC system frequency (ω)	377	rd/s
Inverter-stiff AC system lead angle($\Delta\delta$)	0.1571	rd

Table 3: Parameters for Test II

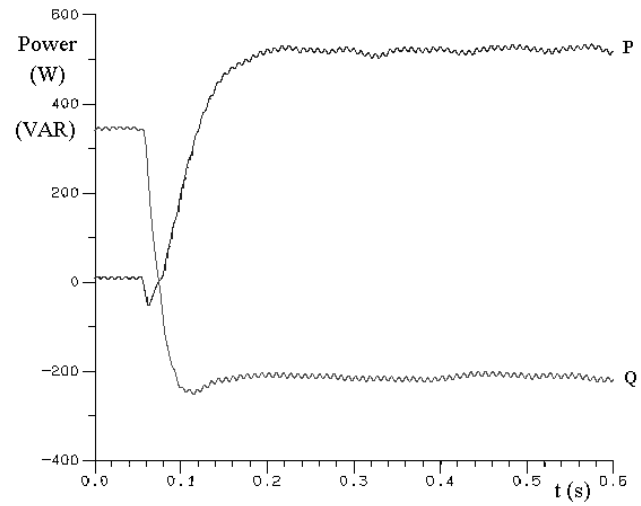

Figure 13: Test II - Active and Reactive Power

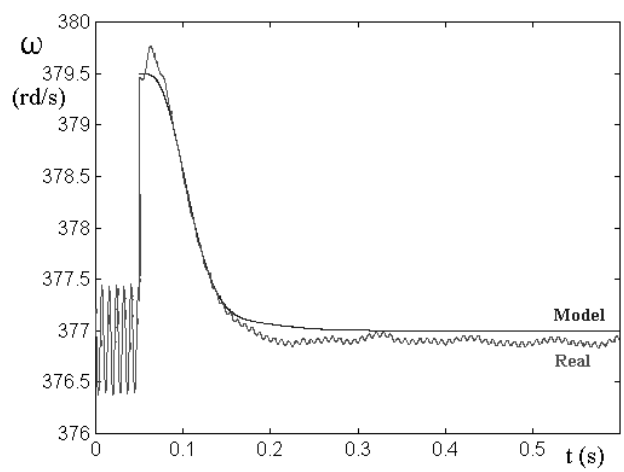

Figure 14: Test II - Real and Model Inverter Frequency

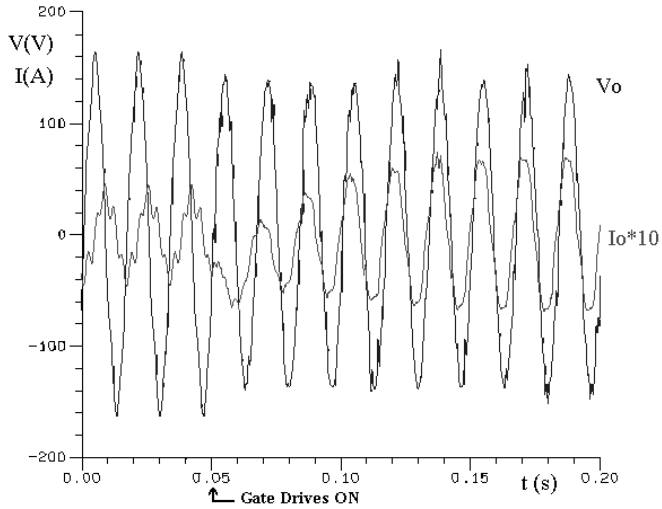

Figure 15: Test II - Inverter Ouput Voltage and Current

Variable	Value	Unit
Line impedance	$0.5+j3.02$	Ω
Cut-off frequency of measuring filter(ω_f)	75.4	rd/s
filter damping factor	0.7	
Frequency voltage droop coefficient (k_p)	0.01	rd/s/W
Frequency voltage droop coefficient (k_v)	0.01	V/VAR
Apparent power in stiff AC system	$500+j250$	VA
Inverter output apparent power	$514+j337$	VA
Stiff AC system voltage (V)	104	V (rms)
Inverter output voltage (E)	114	V (rms)
Stiff AC system frequency (ω)	377	rd/s
Inverter-stiff AC system lead angle($\Delta\delta$)	0.1165	rd

Table 4: Parameters for Test III

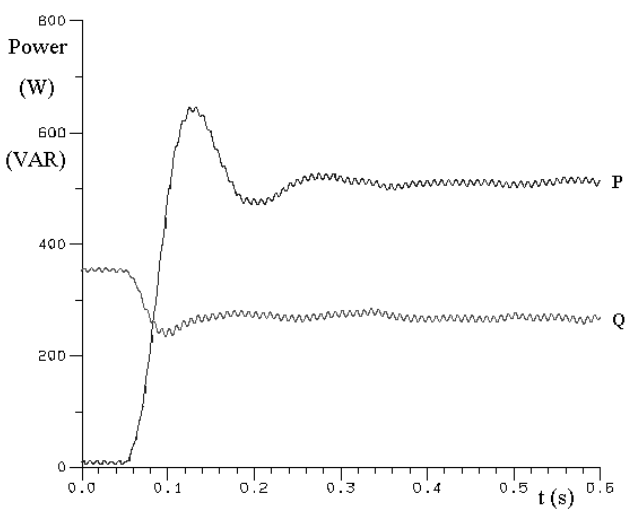

Figure 16: Test III - Active and Reactive Power

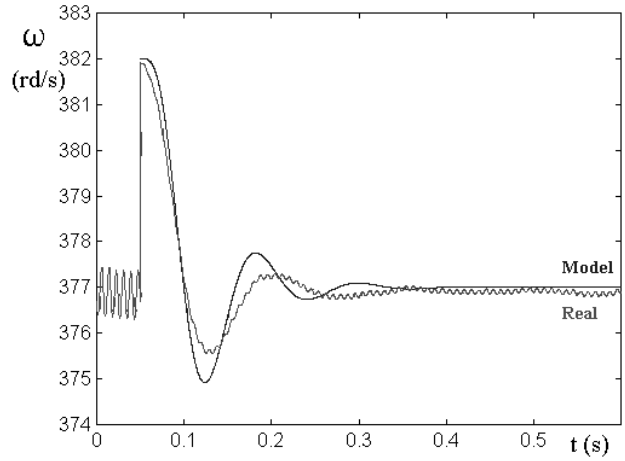

Figure 17: Test III - Real and Model Inverter Frequency

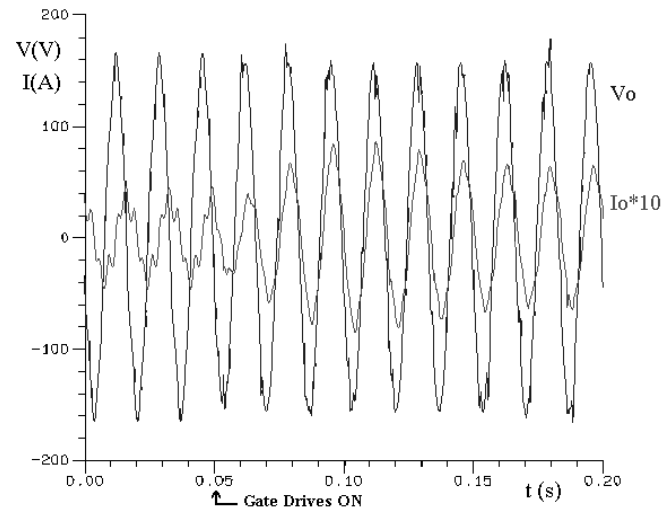

Figure 18: Test III - Inverter Ouput Voltage and Current

Figures 19, 20 and 21 present the results of the test IV, which parameters are shown in table 5. The test IV uses the same k_p and k_v of the test III, but the power reactive flux is negative. The oscillations can be observed in waveforms again.

The experimental results don't completely agree with the small signal model, but it is important to take account that several system parameters were neglected; the frequency in real stiff AC system is not absolutely constant, which causes an error in set point of the active power flux; and the active and reactive power fluxes are not completely decoupled.

6 Conclusion

This paper has presented a small signal analysis for the single inverter connected to stiff AC system, which be-

Variable	Value	Unit
Line impedance	$0.5+j3.02$	Ω
Cut-off frequency of measuring filter(ω_f)	75.4	rd/s
filter damping factor	0.7	
Frequency voltage droop coefficient (k_p)	0.01	rd/s/W
Frequency voltage droop coefficient (k_v)	0.01	V/VAR
Apparent power in stiff AC system	$500-j250$	VA
Inverter output apparent power	$514+j-163$	VA
Stiff AC system voltage (V)	104	V (rms)
Inverter output voltage (E)	100	V (rms)
Stiff AC system frequency (ω)	377	rd/s
Inverter-stiff AC system lead angle($\Delta\delta$)	0.1571	rd

Table 5: Parameters for Test IV

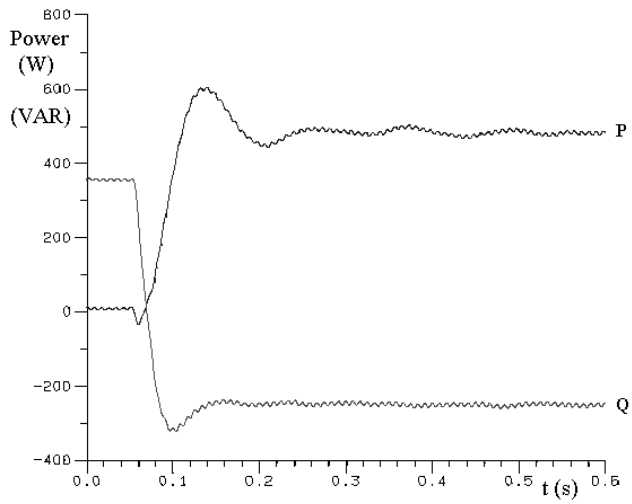

Figure 19: Test IV - Active and Reactive Power

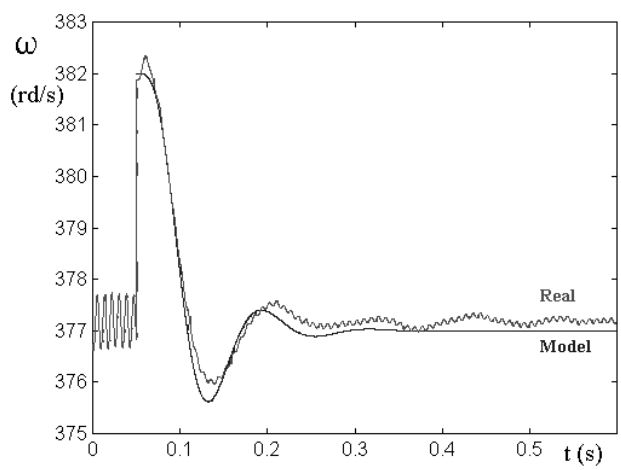

Figure 20: Test IV - Real and Model Inverter Frequency

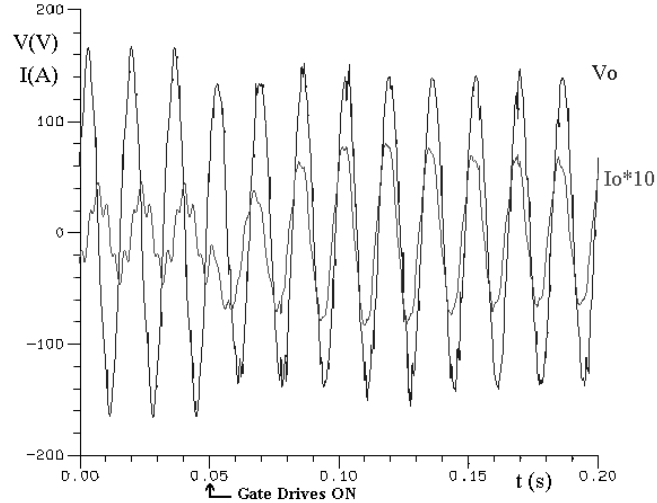

Figure 21: Test IV - Inverter Ouput Voltage and Current

comes stability and performance studies easier. Simulation and experimental results show that the system is well represented by the model obtained through the small signal analysis.

References

[1] Prabha Shankar Kundur. *Power System Stability and Control.* McGraw-Hill, Inc., 1994.

[2] Michael J. Ryan and Robert D. Lorenz. A high performance sine wave inverter controller with capacitor current feedback and "back-emf" decoupling. In *Conf. Rec. IEEE-PESC*, pages 507–513, 1995. Atlanta, GA.

[3] A. Tuladhar, H. Jin, T. Unger, and K. Mauch. Parallel operation of single phase inverter modules with no control interconnections. In *APEC'97*, volume 1, pages 94–100, February 1997. Atlanta, GA.