Understanding Parameter Estimation in Induction Machines

L.A.S. Ribeiro†(*), C.B. Jacobina‡, A.M.N. Lima‡
†Centro Federal de Educação Tecnológica do Maranhão
Av. Getúlio Vargas, 4, Monte Castelo - São Luís, MA - Brazil; 65030 - 810
Email: lantonio@cefet-ma.br;
‡Departamento de Engenharia Elétrica, Universidade Federal da Paraiba

Abstract

This paper studies the problems associated with parameter estimation of the induction machines using the transfer function i_s/v_s. These problems are analyzed through sensitivity and frequency response functions. Based on this study, signal and models to estimate the parameters of the machine are proposed. The discrete time parameter estimation models are written in the δ operator specifically because it provides good numerical properties at high sampling rates, where the discrete time model approaches its continuous time equivalent. This feature permits the direct estimation of the continuous time parameters. The experimental results obtained with the proposed estimation procedure are presented and demonstrate that it is possible to map the parameters in terms of the operating conditions of the induction machine.

I. INTRODUCTION

Parameter and speed estimation in induction machines is a research topic of wide interest for many reasons, including diagnostic and tuning of field oriented drives. But in these applications it is necessary the knowledge of the physical parameters. Unfortunately, mainly due to rapid development in digital electronics and computer technology, most of the parameter estimation problems are discretized for compatibility with digital computers . Then, the physical parameters are calculated from the coefficients estimated in the discrete time domain. Important accuracy problems can arise due to the interaction of finite resolution of analog-to-digital converters and the sampling rate. This is a very important issue, specially when there is not *a priori* knowledge about the system. Slow and relatively fast sampling rates as compared to the dynamic of the system must be avoided [1]. To avoid resolution problems due to the sampling rate, the δ operator is used in this paper for the characterization of the discrete time parameter estimation models. This operator provides good numerical properties at high sampling rates, where the discrete time model approaches its continuous time equivalent. This feature permits the direct estimation of the continuous time parameters.

The use of parameter estimation techniques in the characterization of an induction machine using the transfer function i_s/v_s has been reported by many research teams [2],[3],[4],[5],[6]. These reports do not show clearly the numerical problems associated with the parameter estimation using this approach. This paper analyzes the problems associated with the parameter estimation of induction motors, and propose a signal and a procedure that can be used to estimate the parameters with better numerical properties. The method permits the estimation of l_s, τ_r, and the speed with *a priori* knowledge of r_s, and σl_s. The main feature of the proposed technique is the possibility to estimate the parameters by an almost sinusoidal signal, and without the knowledge of the machine speed. This last feature is an advantage in relation to a former paper of the authors [7].

In the first part of the paper, the induction machine model is described and the problems associated with the estimation of its electrical parameters are analyzed. Then, it is proposed a procedure to estimate the parameters and speed that incorporate some *a priori* knowledge of the machine. Finally, experimental results show the feasibility of the proposed procedure.

II. AC DRIVE SYSTEM

The characterization of the machine as discussed in this paper assumes that the induction machine is part of a microcomputer AC drive system. The drive system is composed of a static power converter, a three phase induction machine coupled to a mechanical load (simulated here by a separately excited direct current generator supplying a resistive load) and a microcomputer whose software controls the overall functioning of the drive. The generation of the command signals for the converter, the data acquisition, and the control laws are implemented around a microcomputer-based platform that is equipped with the appropriate plug-in boards and sensors. The system is shown in Fig. 1.

III. MACHINE DYNAMIC MODEL

In this paper the standard mathematical description of the induction machine is as follow: $j = \sqrt{-1}$, $\mathbf{i}_s = i_{sd} + ji_{sq}$, and $\mathbf{v}_s = v_{sd} + jv_{sq}$. Where $\mathbf{i}_s$, and $\mathbf{v}_s$ are the stator current, and the input stator voltage vectors, respectively. Based on the dynamic model of the machine, the transfer function i_s/v_s can be written as the following [8]:

$$\frac{i_s}{v_s} = \frac{\frac{1}{\sigma l_s}s + \frac{1}{\sigma l_s}(\frac{1}{\tau_r} - j\omega_r)}{s^2 + (\frac{r_s + l_s/\tau_r}{\sigma l_s} - j\omega_r)s + \frac{r_s}{\sigma l_s}(\frac{1}{\tau_r} - j\omega_r)} \quad (1)$$

with s representing the Laplace transform. The quantity represented by ω_r is the machine speed. The electrical

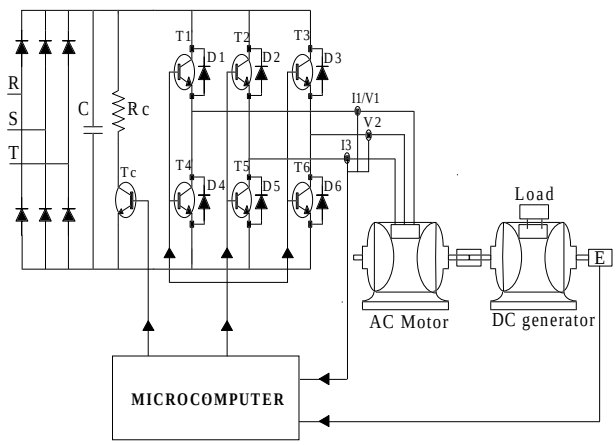

Fig. 1 AC drive system configuration

parameters l_s, l_r and l_m stand for the self inductance of the stator winding, the self inductance of the rotor, and the mutual inductance between stator and rotor. The resistances of the stator and rotor are denoted by r_s and r_r, while $\sigma = 1 - l_m/(l_s l_r)$ and $\tau_r = l_r/r_r$ are the leakage factor and rotor time constant.

Re-writing (1) in the time domain gives (2). This expression is useful for estimation of the machine parameters using a linear regression model.

$$\begin{aligned}\frac{d^2\mathbf{i}_s}{d^2t} &= -\left(\frac{r_s + l_s/\tau_r}{\sigma l_s} - j\omega_r\right)\frac{d\mathbf{i}_s}{dt} - \frac{r_s}{\sigma l_s}\left(\frac{1}{\tau_r} - j\omega_r\right)\mathbf{i}_s \\ &\quad + \frac{1}{\sigma l_s}\frac{d\mathbf{v}_s}{dt} + \frac{1}{\sigma l_s}\left(\frac{1}{\tau_r} - j\omega_r\right)\mathbf{v}_s \qquad (2)\end{aligned}$$

The use of system identification techniques as proposed in this paper is based on a discrete-time representation of the induction machine behavior (2). The discrete-time model of an induction machine may be derived from (2) by using z-transform or γ-transform techniques. In this work the γ-transform is selected specifically because it provides good numerical properties at high sampling rates [9], where the discrete model approaches the continuous equivalent. The discrete-time model obtained from (2) by using the γ-transform is represented by:

$$\delta^2\mathbf{i}_s = F_1\delta\mathbf{i}_s + F_0\mathbf{i}_s + H_1\delta\mathbf{v}_s + H_0\mathbf{v}_s, \qquad (3)$$

where $F_1 = F_{1a} + jF_{1b}$, $F_0 = F_{0a} + jF_{0b}$, $H_1 = H_{1a} + jH_{1b}$, and $H_0 = H_{0a} + jH_{0b}$ are the discrete time parameters, δ is the delta operator, defined as $\delta x(t) = [x(t+h) - x(t)]/h$, and h is the sampling period. It is assumed that the stator voltage vector $\mathbf{v}_s$ is supplied to the machine through a zero order hold device and the speed ω_r is assumed to remain constant during the sampling period.

If the sampling rate is sufficiently high, the discrete-time system (3) can be approximated by its continuous-time equivalent (2). In terms of real and imaginary components of $\mathbf{i}_s$ and $\mathbf{v}_s$, this continuous-time model may be rewritten as

$$\begin{aligned}\begin{bmatrix}\delta^2 i_{sd}\\ \delta^2 i_{sq}\end{bmatrix} &= \begin{bmatrix}-\frac{r_s + l_s/\tau_r}{\sigma l_s} & -\omega_r \\ \omega_r & -\frac{r_s + l_s/\tau_r}{\sigma l_s}\end{bmatrix}\begin{bmatrix}\delta i_{sd}\\ \delta i_{sq}\end{bmatrix} + \\ &\begin{bmatrix}-\frac{r_s}{\sigma l_s \tau_r} & -\frac{\omega_r r_s}{\sigma l_s} \\ \frac{\omega_r r_s}{\sigma l_s} & -\frac{r_s}{\sigma l_s \tau_r}\end{bmatrix}\begin{bmatrix}i_{sd}\\ i_{sq}\end{bmatrix} + \begin{bmatrix}\frac{1}{\sigma l_s} & 0 \\ 0 & \frac{1}{\sigma l_s}\end{bmatrix}\begin{bmatrix}\delta v_{sd}\\ \delta v_{sq}\end{bmatrix} \\ &+ \begin{bmatrix}\frac{1}{\sigma l_s \tau_r} & \frac{\omega_r}{\sigma l_s} \\ -\frac{\omega_r}{\sigma l_s} & \frac{1}{\sigma l_s}\end{bmatrix}\begin{bmatrix}v_{sd}\\ v_{sq}\end{bmatrix} \qquad (4)\end{aligned}$$

This also represents that $\delta x(t) \simeq dx(t)/dt$.

To estimate the parameters using the Least Squares (LS) algorithm it is necessary to rewrite the model (4) in the form of a linear regression such as

$$y(t) = ,(t)\theta \qquad (5)$$

where $y(t)$, $,(t)$, and θ are the prediction vector, the regression matrix, and the parameter vector, respectively. For example, observe that (4) may be rewritten as linear regression model simply by defining

$$\begin{aligned} y(t) &= \begin{bmatrix}\delta^2 i_{sd}\\ \delta^2 i_{sq}\end{bmatrix}, \\ ,(t) &= \begin{bmatrix}-\delta i_{sd} & -\delta i_{sq} & -i_{sd} & -i_{sq} & \delta v_{sd} & v_{sd} & v_{sq} \\ -\delta i_{sq} & \delta i_{sd} & -i_{sq} & i_{sd} & \delta v_{sq} & v_{sq} & -v_{sd}\end{bmatrix}, \\ \theta &= \begin{bmatrix}\frac{r_s l_r + r_r l_s}{\sigma l_s l_r} & \omega_r & \frac{r_s}{\sigma l_s \tau_r} & \frac{r_s \omega_r}{\sigma l_s} & \frac{1}{\sigma l_s} & \frac{1}{\sigma l_s \tau_r} & \frac{\omega_r}{\sigma l_s}\end{bmatrix} \qquad (6)\end{aligned}$$

The Least Square algorithm to calculate θ can be found in [9].

IV. ESTIMATION MODELS

The simultaneous estimation of r_s, σl_s, l_s, τ_r, and ω_r in (6) requires the use of special excitation signals. Nevertheless, the r_s is estimated with such error and variance that prohibits its use in the subsequent estimation and control procedures [2], [3], [4]. It is possible to explain this result through sensitivity functions that are derived from (3), and are given by (7). With these sensitivity functions it is possible to determine in which region the parameters are better estimated.

$$\frac{\partial y(t)}{\partial \theta_i}\frac{\theta_i}{y(t)} \qquad (7)$$

The frequency response function (FRF) of the sensitivity (7) with respect to r_s is shown in Fig. 2a. The FRF is plotted for two different speeds: $\omega_r = 0$ rad/s and $\omega_r = 2\pi 60$ rad/s. The bigger the sensitivity of $y(t)$ with respect to a specific parameter is, the bigger its influence will be on the prediction vector $y(t)$, and consequently its estimation will be easier. This means that a given parameter will be better estimated at the frequencies where its FRF (see Fig. 2a) is high.

The conclusion that can be drawn by analyzing Fig. 2a is that it is possible to estimate r_s only with low frequency excitation waveforms when $\omega_r = 0$ rad/s. This is a known result since the resistance determines the stead state gain [6]. However, if $\omega_r = 2\pi 60$ rad/s a good estimation can be obtained for excitation frequencies below $2\pi 10$ rad/s. Again, a low frequency excitation waveform must be used to determine this parameter.

In normal operating conditions the slip is in the range 1% to 10% and consequently the fundamental frequency

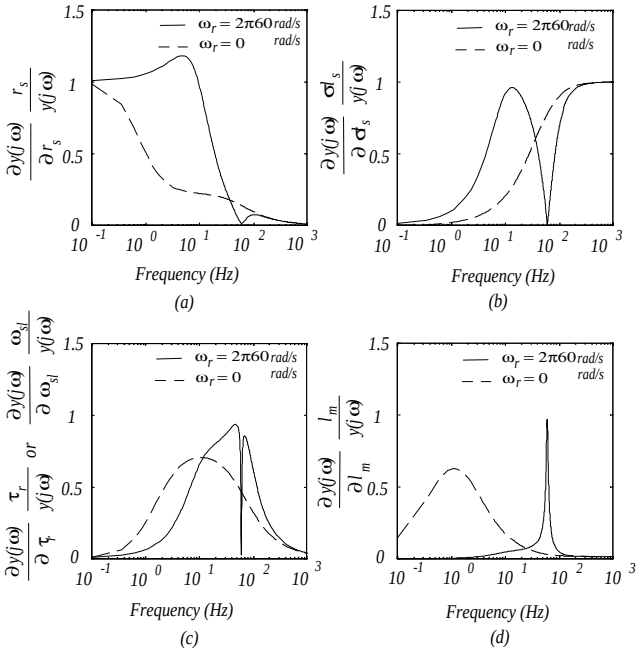

Fig. 2 FRF of the sensitivity (7) with respect to: a) r_s; b) σl_s; c) τ_r or ω_{sl}; d) l_m

of the excitation waveform is around ω_r. As it can be observed from Fig. 2a, the FRF is close to zero around $\omega \approx \omega_r = 2\pi 60$ rad/s and then it is almost impossible to estimate r_s, unless a low frequency component or a dc bias be superimposed to the excitation signal. The same conclusion is valid for the case where $\omega_r = 0$ rad/s. Thus, it is interesting to propose a model to estimate solely r_s and remove it from the parametric vector (6). This should improve the accuracy of the estimation of the other parameters.

Fig. 2b shows the FRF of the sensitivity (7) with respect to σl_s. In the normal operating region (slip in the range of 1% to 10%), it is not possible to estimate this parameter. However, for high frequencies this parameter has a strong effect on the sensitivity and its estimation is completely possible. This is due to the skin effect proven at high frequencies. In this region almost all the machine flux is leakage flux. In [5] the estimation of σl_s with model (6) and a six-step waveform was shown to be very good. The high frequency harmonics of the six-step waveform provide persistent excitation to estimate σl_s. By considering that the fundamental excitation does not provide almost any information about σl_s, it is interesting to propose a specific procedure for estimating σl_s and remove it from θ (6). As it has been mentioned for the case of r_s, this should improve the accuracy of the estimation of the other parameters.

The FRF of the sensitivity (7) with respect to τ_r and ω_{sl} are shown in Fig. 2c. These FRF's are exactly equals. This lead to a known result: it is impossible to distinguish τ_r and ω_{sl} at steady state. The second conclusion is that, at synchronous speed, τ_r, and ω_{sl} can not be estimated. This is due to the fact that the rotor and mechanical states are not observed at the synchronous speed. Third, all the information of these parameters are present in the fundamental excitation.

The FRF of the sensitivity (7) with respect to l_m is shown in Fig. 2d. It can be observed that it is possible to estimate l_m only around the synchronous frequency. It is exactly at this frequency that l_m is better identified. At the synchronous frequency the slip is zero and the induction machine is basically an RL load.

Furthermore, Figs. 2a - d show that as the speed decreases the parameters are worse estimated, except for r_s. Another conclusion is that the fundamental excitation has information only about the rotor time constant τ_r, the speed ω_r, and the magnetize inductance l_m (and l_s).

In this paper, τ_r, l_s, and ω_r are estimated based on *a priori* knowledge of r_s and σl_s. The parameter estimation problem is solved by using the standard least squares technique (LS) [9]. The models used in this paper are described in the following sections.

A. Model A: estimation of ω_r - r_s and σl_s known

If r_s and σl_s are known, it is possible to derive a model that should provide a better estimate of τ_r, l_s, and ω_r when compared to the estimates obtained with model (6) if appropriated excitation waveforms are selected. The improvement in the accuracy of the estimates is due mainly to the better numerical conditioning of the covariance matrix of the LS algorithm. However, if sinusoidal waveforms are used for the stator voltages only the speed can be estimated with this model, although the estimation has a large variance. The model is the following:

$$\begin{aligned} y(t) &= \begin{bmatrix} \delta^2 i_{sd} - \frac{1}{\sigma l_s}\delta u_{sd} \\ \delta^2 i_{sq} - \frac{1}{\sigma l_s}\delta u_{sq} \end{bmatrix}, \\ ,(t) &= \begin{bmatrix} -\frac{1}{\sigma l_s}\delta i_{sd} & \delta i_{sq} + \frac{1}{\sigma l_s}u_{sq} & \frac{1}{\sigma l_s}u_{sd} \\ -\frac{1}{\sigma l_s}\delta i_{sq} & \delta i_{sd} - \frac{1}{\sigma l_s}u_{sd} & \frac{1}{\sigma l_s}u_{sq} \end{bmatrix}, \\ \theta &= \begin{bmatrix} \frac{l_s}{\tau_r} & \omega_r & \frac{1}{\tau_r} \end{bmatrix}^T \end{aligned} \tag{8}$$

where $u_{sd} = v_{sd} - r_s i_{sd}$, and $u_{sq} = v_{sq} - r_s i_{sq}$.

B. Model B: Estimation at standstill - r_s and σl_s known

This model should be employed when the machine is at standstill. In this case, the terms involving ω_r equals zero and the model (9) is obtained from (4).

$$\begin{aligned} y(t) &= \begin{bmatrix} \delta^2 i_{sd} & \delta^2 i_{sq} \end{bmatrix}^T, \\ ,(t) &= \begin{bmatrix} -\delta i_{sd} & u_{sd} & \delta u_{sd} \\ -\delta i_{sq} & u_{sq} & \delta u_{sq} \end{bmatrix}, \\ \theta &= \begin{bmatrix} \frac{l_s}{\sigma l_s \tau_r} & \frac{1}{\sigma l_s \tau_r} & \frac{1}{\sigma l_s} \end{bmatrix}^T \end{aligned} \tag{9}$$

V. EXPERIMENT DESIGN

The design of the experiment is a crucial problem in parameter estimation. In the present paper the experiment design is considered with respect to each proposed estimation model.

In this paper it is considered that r_s and σl_s are previously estimated. The stator resistance can be estimated by superimposing a dc level to the fundamental excitation as proposed in [10] or by superimposing an homopolar signal to the fundamental excitation as discussed in [11]. The stator transient inductance σl_s can be estimated using the complete model of the machine (6) or by using high frequency signal as proposed in [7].

For the estimation of τ_r, l_s, and ω_r with Model A a signal composed of the fundamental excitation plus a sinusoid with a magnitude of 5 % of the fundamental excitation and a frequency 15 % lower than the fundamental frequency was used. The choice of this frequency was based on he FRF of the sensitivity function (7) with respect to τ_r. As can be seen in Fig. 2c the FRF has its maximum value at a frequency approximately 15 % lower than the fundamental frequency. This result is true at almost all fundamental frequencies except at zero.

In the case of the zero speed tests (Model B), no mechanical means have been employed to keep the machine at standstill. This is achieved by using a signal which produces an almost zero average electro-magnetic torque, and the machine stays at standstill .

The signals (v_{sd}, v_{sq}) and (i_{sd}, i_{sq}) are obtained by measuring two of the three phase quantities. The signals are filtered (anti-aliasing) and converted through four 10 bit A/D converters. The derivative terms δv_{sd}, δv_{sq}, δi_{sd}, δi_{sq}, $\delta^2 v_{sd}$ $\delta^2 v_{sq}$, $\delta^2 i_{sd}$ and $\delta^2 i_{sq}$ are obtained by digital filtering. Four identical third order digital filters were employed for each directly measured stator quantity.

VI. PARAMETER SENSITIVITY

The model used in this work to estimate τ_r, l_s, and ω_r depends on *a priori* knowledge of r_s and σl_s. So it is important to analyze the parameter sensitivity of Model A with respect to variations in r_s and σl_s. Fig. 3 shows the parameter sensitivity of Model A with respect to variations in r_s. The parameter sensitivity is shown for two different fundamental frequencies: 5 Hz and 60 Hz. For each frequency the slip is varied in the range from 1% to 10%, and the value of r_s used in Model A is 30% lower than the actual value of the machine used in the simulation results. The figures show the error of each estimated parameter of Model A with respect to the true parameter of the machine. It can be noted form Fig. 3a, and Fig. 3d that even if a wrong value of r_s is used the estimation errors in l_s and ω_r remain within acceptable margins (less than 3%) independent of the fundamental frequency (speed) and the slip. The τ_r is estimated with unacceptable error at low frequency (Fig. 3b), especially at low slip. This is expected since it is hard to estimate ω_r at low slip (see Fig. 2c). It must be noted that this very high error occur because the error in r_s used in Model A is 30%. It is clear that if r_s is estimated with precision (error lower than 10%) it is still possible to estimate τ_r.

Fig. 4 shows the parameter sensitivity of Model A with respect to variations in σl_s. The parameter sensitivity is negligible in the frequency and slip ranges shown in the

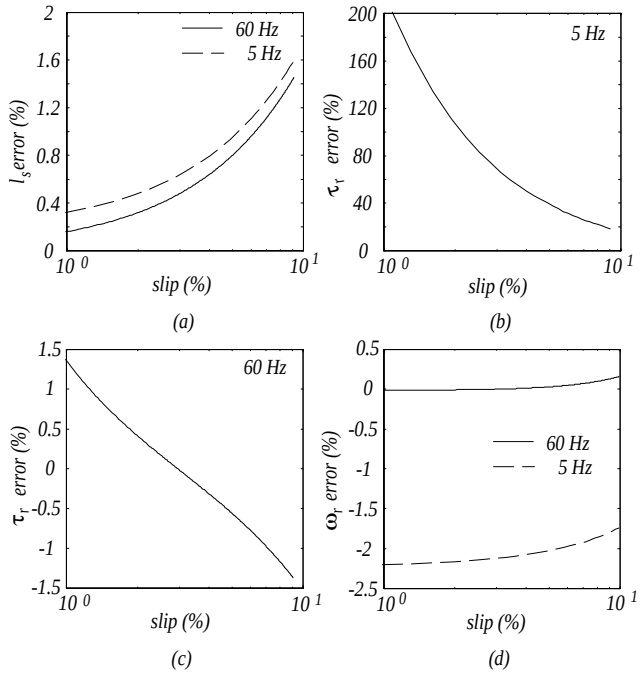

Fig. 3 Sensitivity of Model A with respect to variation in r_s - machine 1: a) l_s ; b) τ_r ; c) τ_r; d) ω_r

Fig. 4. Furthermore, the parameter sensitivity of τ_r (Fig. 4b) is bigger than the parameter sensitivity of the other two parameters. The main problem of uncertainty in σl_s is in the field weakening range. This is the region where σl_s has a strong effect on the Model A.

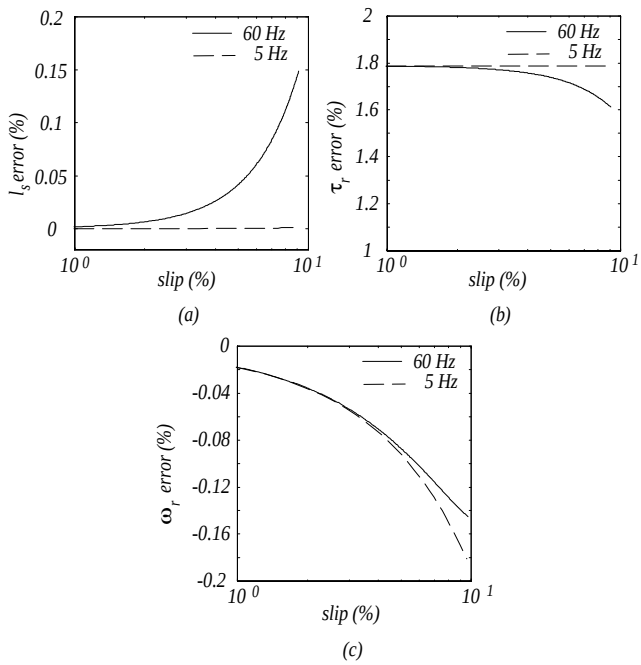

Fig. 4 Sensitivity of Model A with respect to variation in σl_s - machine 1: a) l_s ; b) τ_r ; c) ω_r

VII. SIMULATION AND EXPERIMENTAL RESULTS

The proposed models were studied previously by digital simulation where it was assumed that the measurements were corrupted by an additive noise term. Two machines were used in the results. The parameters obtained through the classic procedures, e.g. locked rotor and no-load test (rbv) [12], are presented in Table 1. Machine 1 (3 hp, 220 V, 1700 rpm) was used in the experimental tests, and machine 2 (3 hp, 220 V, 1710 rpm) was used in the simulation studies [13].

	Nameplate data and parameters					
	I (A)	r_s (Ω)	r_r (Ω)	l_m (H)	l_{ls} (H)	l_{lr} (H)
M1	5.8	1.8	2.99	0.1168	0.0111	0.0111
M2	5.8	0.435	0.82	0.0693	0.0002	0.0002

Tabel 1 Nameplate dada, and parameters of the machines used in the experimental results

Fig. 5 shows the simulation results of the estimation of τ_r, l_s, and ω_r using the fundamental excitation + sinusoid at a frequency 15% lower. The machine was running at rated load (slip = 5 %). As can be shown the errors are lower than 5 %. The errors at no-load are lower than the errors at rated load. These results can be seen in Fig. 6.

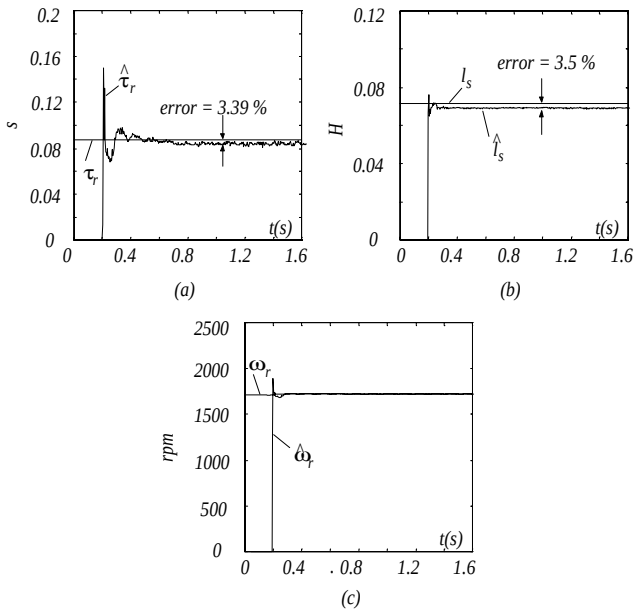

Fig. 5 Estimation with Model A at slip = 5 % - machine 1: a) τ_r (s); b) l_s (H); c) ω_r (rpm)

The estimation algorithm employed to process the experimental data was the recursive least squares with forgetting factor. The sampling time was set to $h = 50\mu s$ and the forgetting factor to $\lambda = 0.999$. Fig. 7 shows the experimental results obtained with Model B at zero speed. The variances and the transient estimation in this case are better than the ones obtained when r_s is also estimated [5].

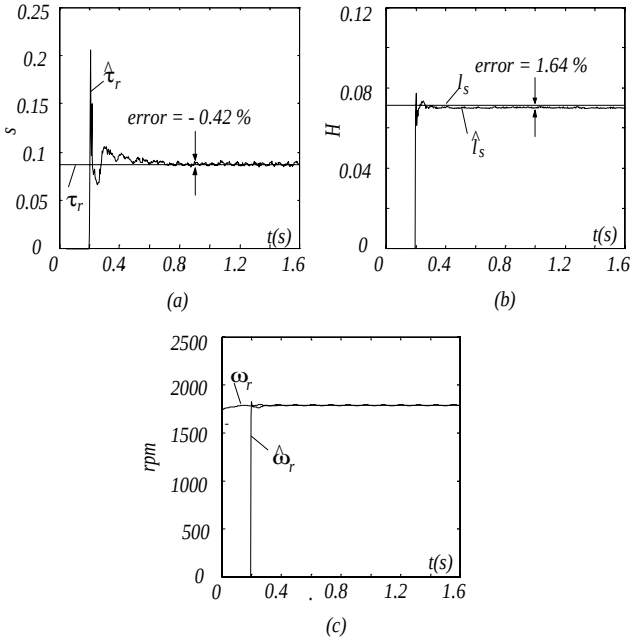

Fig. 6 Estimation with Model A at no-load - machine 1: a) τ_r (s); b) l_s (H); c) ω_r (rpm)

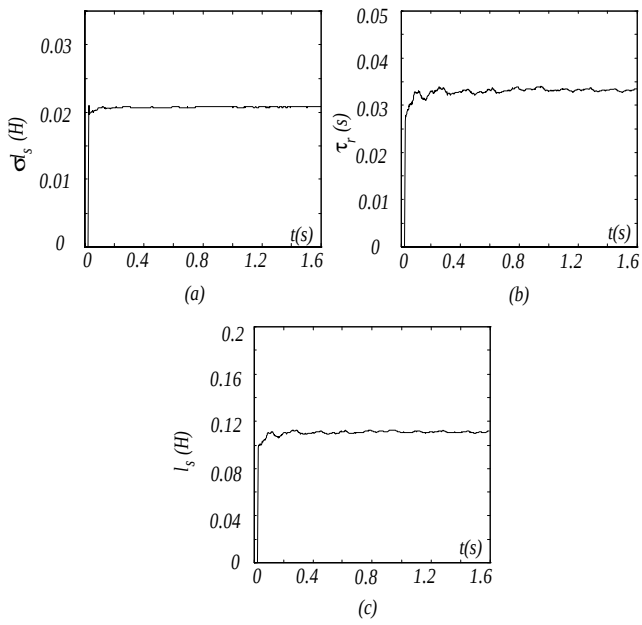

Fig. 7 Experimental esstimation with Model B at $\omega_r = 0$ - machine 1: a) σl_s (H); b) τ_r (s); c) l_s (H)

VIII. CONCLUSION

This paper has discussed the problems associated with the parameter estimation of the induction machine by using the transfer function i_s/v_s. The conclusions of this study are:

1. The fundamental excitation has information only about τ_r, ω_r, and l_m (and l_s);
2. As the speed decreases the parameters are worse estimated, except for r_s;

3. All the information about r_s is presented at low and zero frequencies;
4. Incorporating *a priori* knowledge of r_s and σl_s on the estimation model improves the estimation of the other parameters (τ_r , l_s , and ω_r);
5. The effect that the uncertainty in r_s and σl_s has on the parameter estimation is negligible, except for the estimation of τ_r at low speed;
6. By using an almost sinusoidal signal it is possible to estimate τ_r, l_s, and ω_r at all operating conditions.

REFERENCES

[1] D. E. Borgard, G. Olsson, R. D. Lorenz, "Accuracy issues for parameter estimation of field oriented induction machine drives," in *Conf. Rec. IAS*, pp. 593–600 1994.

[2] M. Véles-Reyes, K. Minami, G. C. Verghese, "Recursive speed and parameter estimation for induction machines," in *Conf. Rec. IAS*, pp. 607–611 1989.

[3] J. Stephan, M. Bodson, J. Chiasson, "Real-time estimation of the parameters and fluxes of induction motors," *IEEE Trans. Ind. Applicat.*, 30, no. 3, 746–759 May/June 1994.

[4] L. A. S. Ribeiro, C. B. Jacobina, A. M. N. Lima, "Dynamic estimation of the induction machine parameters and speed," in *Conf. Rec. PESC*, pp. 1281–1287 1995.

[5] L. A. S. Ribeiro, C. B. Jacobina, A. M. N. Lima, "Parameter and speed estimation of induction machine based on dynamic models," in *Conf. Rec. EPE*, pp. 1496–1501 1995.

[6] R. J. A. Gorter, P. P. J. V. den Bosh, S. Weiland, "Simultaneous estimation of induction machine parameters and velocity," in *Conf. Rec. PESC*, pp. 1295–1301 1995.

[7] L. A. S. Ribeiro, C. B. Jacobina, A. M. N. Lima, "Real-time estimation of the electrical parameters of an induction machine using sinusoidal PWM voltage wavwform," in *Conf. Rec. IAS*, pp. 746–752 1997.

[8] L. A. S. Ribeiro, C. B. Jacobina, A. M. N. Lima, "Linear parameter estimation for induction machines considering the operating conditions," *IEEE Trans. Power Electronics*, 14, no. 1, 62–73 January 1999.

[9] R. Middleton, G. Goodwin, Digital control and estimation. A unified approach, Prentice-Hall, Inc., Englewood Cliffs, New Jersey 1990.

[10] L. A. S. Ribeiro, C. B. Jacobina, A. M. N. Lima, "The influence of the slip and the speed in the parameter estimation of induction machines," in *Conf. Rec. PESC*, pp. 1068–1074 1997.

[11] C. B. Jacobina, J. E. Chaves, A. M. N. Lima, L. A. S. Ribeiro, "A new method for estimating all the electrical parameters of induction machines," in *Conf. Rec. IAS*, pp. 265–270 1998.

[12] I. S. Board, "Electric machinery," in *IEEE Standard Test Procedure for Polyphase Induction Motors and Generators*, Piscataway, New Jersey 1995, IEEE Press.

[13] P. Krause, O. Wasynczuk, S. Sudhoff, Analysis of Electric Machinery, IEEE Press, IEEE, Inc., New York 1990.