Dynamic Capacitance Switching Applied to Asymmetrical Three-Phase Induction Motors - The Load Diversity Condition.

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***Abstract* - This paper presents a behavioral analysis of the asymmetrical induction motor fed by a single phase source. The interesting aspect in this case is the machine behavior when it is applied a dynamic switching along the different operating conditions, in this case different loading conditions. Here is presented the theoretical results for the capacitance switching application in a 2 HP asymmetrical three-phase induction motor. This analysis allow us to obtain conclusive facts on the asymmetrical motor operation when it is applied a capacitance dynamic switching system.**

I. INTRODUCTION

The three-phase asymmetrical induction motor fed by a voltage single-phase system[1][5] has a squirrel-cage rotor and stator where the three-phase windings are located with each phase spatial displaced of 120° and number of turns different in each phase according to design considerations. With the utilization of a capacitance, C_{ap}, connected between phases B and C, as in Figure 1, it will be possible to obtain an operation point in which the motor develops its nominal speed and power. In that case will be necessary to obtain the correct number of turns per phase and the capacitance C_{ap}.

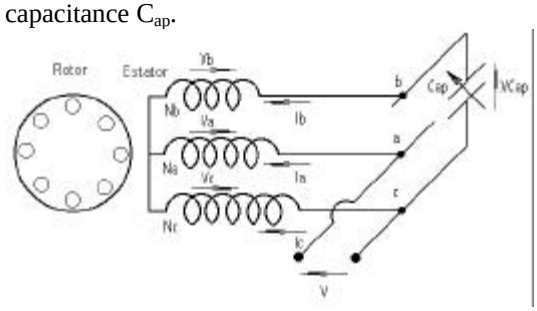

Fig. 1 – Asymmetrical three-phase induction motor schematic diagram.

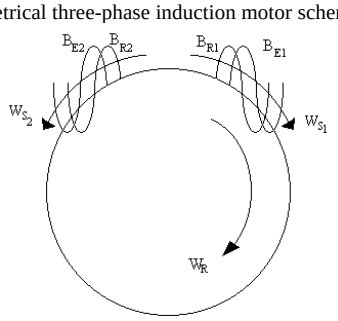

Fig. 2 – Rotational magnetic field representation for the asymmetrical induction motor.

Therefore, the motor phase currents turn to be an unbalanced three-phase system, and as a consequence this motor has unbalanced magnetomotrice force (*mmf*) distribution in each phase. This unbalance allows the decomposition of such phase *mmf* distribution in order to obtain static and rotational magnetic fields distribution, where the later fields has positive and negative sequence. The positive sequence rotational magnetic field, B_{S1}, through the symmetrical three-phase induction machine operating principles, gives also origin indirectly to a rotational magnetic field, B_{R1}, with the same sequence in the machine's rotor. The same happens with the negative sequence magnetic field, B_{S2}, creating B_{R2}. This fact is illustrated in Figure 2, and can be seen clearer that:

- Magnetic fields (B_{S1}, B_{R1}) and (B_{S2}, B_{R2}) have between them a constant angular displacement. Therefore this results in a constant aligning electromagnetic torque;
- The angular position between magnetic fields (B_{S1}, B_{R2}) and (B_{S2}, B_{R1}) change with periodic pattern. The resulting electromagnetic torque in this case is an oscillatory value.

So, the instantaneous value for the electromagnetic torque produced by the induction motor in steady state is oscillatory around the nominal operating point. However, these oscillations for the nominal condition are perfectly acceptable, since they do not cause major problems to the motor normal operating condition[5].

Another important untouched issue in previous discussions is the asymmetrical induction motor with single phase feeding under the effect of variable load. The analysis ranges from no-load to 20% overloaded condition. The main goal here is to do the analysis of the implementation of a dynamic switching capacitance. It is intended here to restrict torque oscillations to a minimum along with a wide range of loading conditions. Theoretical results will be obtained from the asymmetrical three-phase induction motor mathematical model. This takes into consideration the machine operational dynamic conditions.

II. MATHEMATICAL MODELING – ASYMMETRICAL MOTOR.

From Figure 1 can be extracted the following equations:

$$\frac{dV_{Cap}}{dt} = \frac{1}{Cap}.i_{Cap} \qquad (1)$$

$$i_c = -(i_a + i_b) \qquad (2)$$

$$V = V_a - V_c \qquad (3)$$

The voltage and current relationship in an ordinary induction machine working as a generator is represented by the generic equation:

$$v_i = r_i.i_i + \frac{d\,l_i}{dt} \quad (4)$$

where:
v_i , i_i , r_i , l_i – voltage, current, resistance and magnetic flux coupling for an i phase in the generator, respectively;
i index – represents one generator phase (a, b, c in the stator - A, B, C in the rotor).

The magnetic flux coupling is given by the generic equation:

$$l_i = (L_i + L_{ii}).i + \sum_{i \neq j} L_{ij}.i_i \quad (5)$$

where:
$L_i + L_{ii}$ – self inductance for a generator's phase i.
L_{ij} – mutual inductance between generator's phases i and j.
i_i , i_j – instantaneous currents for phases i and j , respectively.

In the three-phase induction generator at Figure 1, the self and mutual inductances are given by the following matrix [L]:

$$[L] = \begin{bmatrix} [L1] \\ [L2] \end{bmatrix} \quad (6)$$

$$[L1] = \begin{bmatrix} A_{11} & A_{12} & 0 & A_{13} & A_{14} & A_{15} \\ A_{12} & A_{22} & 0 & A_{23} & A_{24} & A_{25} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

$$[L2] = \begin{bmatrix} A_{13} & A_{23} & 0 & A_{33} & 0 & 0 \\ A_{14} & A_{24} & 0 & 0 & A_{33} & 0 \\ A_{15} & A_{25} & 0 & 0 & 0 & A_{33} \end{bmatrix} \quad (8)$$

where:

A_{11}=2(L_{SS}-M_{SS});
A_{12}= - (L_{SS}+M_{SS});
A_{13}=M_{SS}-M_{SR};
A_{14}=M_{SS}-M_{SR};
A_{15}=0;
A_{22}= -2M_{SS};
A_{23}=0;
A_{24}=0;
A_{25}=0;
A_{33}=L_{RR}.

L_{SS} - represents the values of self inductances in each stator phase (a, b, c);
L_{RR} - represents the values of self inductances in each rotor phase (A, B, C) referred to the stator;
M_{SS} - represents the values of mutual inductances between stator phases (a, b, c);
M_{SR} - represents the values of mutual inductances between stator phases (a, b, c) and rotor phases (A, B, C).

L_{aa}=L_{bb}=L_{cc};
L_{ab}=L_{ac}=L_{bc};
M_{aA}=M_{bB}=M_{cC}=M.cosθ
M_{aB}= M_{bC}= M_{cA}=M.cos(θ - 2π/3)
M_{aC}= M_{bA}= M_{cB}=M.cos(θ + 2π/3)

Mechanical equations for the three-phase induction generator are given by:

$$\frac{dW_R}{dt} = \frac{1}{J}.(T_b - T_m) \quad (9)$$

$$\frac{dq}{dt} = \frac{p}{2}.W_R \quad (10)$$

$$T_m = \frac{p}{4}.[I]^T.\left[\frac{\partial L}{\partial q}\right].[I] \quad (11)$$

where:

W_R – rotor mechanical angular speed;
θ - rotor electrical angle displacement;
p – number of poles;
T_m – generator electromagnetic torque;
T_b - turbine torque.

III. Unbalancing factor

As can be seen in Figure 2, the asymmetrical motor is operating under the influence of rotational magnetic fields which rotates in opposite directions. Therefore, they create electromagnetic torque also in opposite directions. As a consequence is obtained a resulting electromagnetic torque that pulsates and its average value oscillates around the operating point. A new parameter give us the relationship between the two electromagnetic torque, which also express respectively, the relationship between the positive and negative sequence *mmf's*. This relationship is named *Unbalancing Factor* (F), which will be the reference to choose the best turns number set in each phase and the capacitance to be used.

The unbalancing factor is given by:

$$\dot{F} = \frac{F\dot{mm}_{a2}}{\dot{F}\,mm_{a1}} \quad (12)$$

where:

$$\dot{F}\,mm_{a1} = Na.\dot{I}\,a_p \quad (13)$$

$$\dot{F}\,mm_{a2} = Na.\dot{I}\,a_n \quad (14)$$

By substituting equations (13) and (14) in (12) the unbalancing factor is obtained in this case through the relationship between positive and negative sequence currents at phase *a*. Working to leave the unbalancing factor[4], as a function of C_{ap} and the relationships between the winding turns number for phases B and C with A, and calling them "b" and "c", we have:

$$\dot{F} = \frac{\dot{Ia}_n}{\dot{Ia}_p} = \frac{Z_1 - \dot{Z}o.a_1 + j.\frac{1}{b}.Xc.(a_1 - a^2)}{Zo.a_2 - \dot{Z}_2 + j.\frac{1}{b}.Xc.(a - a_2)} \quad (15)$$

where:

$$a = \cos 120^o + j.\text{sen}120^o \quad (16)$$

$$a_1 = \frac{1 + \frac{1}{b}.a^2 + \frac{1}{c}.a}{1 + \frac{1}{b} + \frac{1}{c}} \quad (17)$$

$$a_2 = \frac{1 + \frac{1}{b}.a + \frac{1}{c}.a^2}{1 + \frac{1}{b} + \frac{1}{c}} \quad (18)$$

$$\dot{Z}o = (b - c).\dot{Z}sa \quad (19)$$

$$\dot{Z}_1 = (a^2.b - a.c).\dot{Z}sa_p \quad (20)$$

$$\dot{Z}_2 = (a.b - a^2.c).\dot{Z}sa_n \quad (21)$$

where:
Zsa, Za_p e Za_n are the zero, positive and negative sequence impedance, obtained from the ordinary induction motor equivalent circuit, and similarly Zo, Z_1 e Z_2 are respectively the same impedance for the asymmetrical induction motor.

III. Capacitance dynamic switching

The capacitance dynamic switching consists in the connection of capacitors as the load changes. It means that for every load variation in the motor axis a different capacitor must be switched. For every load regimen a different *mmf* distribution in the machine's air-gap is established, and consequently the internal unbalance changes, this turns to be a favorable environment for pulsating electromagnetic torque. The objective of this switching is to promote a reduction in the internal unbalance.

At reference [5] was demonstrated that a good asymmetrical motor performance at nominal conditions was obtained with the utilization of a fixed capacitor in steady state, but for half load condition, as an example, do not promote a good effect. The fixed capacitance becomes, for half load, the cause of torque oscillations larger than that at nominal condition. Therefore, following this philosophy, a different capacitance will be implemented for every load condition.

IV. Digital simulation

In order to verify the electromagnetic torque oscillation effects for diverse load conditions, an asymmetrical induction motor fed by a single-phase source with the following characteristics will be used:

Nb = 0.60 Na; Nc = 1.40 Na; C_{ap} (nominal) = 40μF

TABLE I
EQUIVALENT CIRCUIT PARAMETERS FOR THE ASYMMETRICAL THREE-PHASE INDUCTION MOTOR.

Stator Resistance	3.80±0.03Ω
Rotor Resistance (referred to the stator)	3.01±0.03Ω
Locked Rotor Resistance (referred to the stator)	3.10±0.03Ω
Phase Magnetization Reactance	75.15±0.70Ω

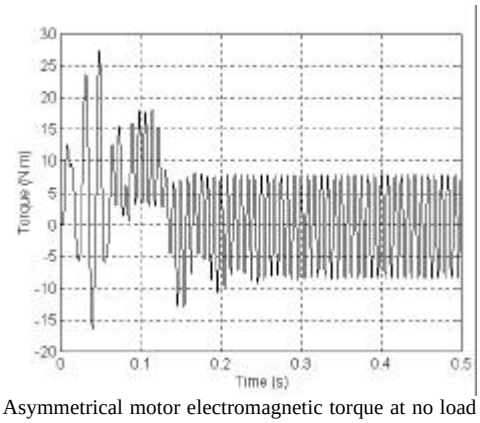

Fig. 3 – Asymmetrical motor electromagnetic torque at no load condition – C_{ap} = 40μF.

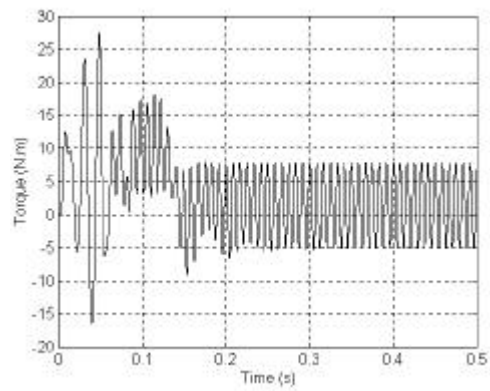

Fig. 4 – Asymmetrical motor electromagnetic torque with 20% of load – C_{ap} = 40μF.

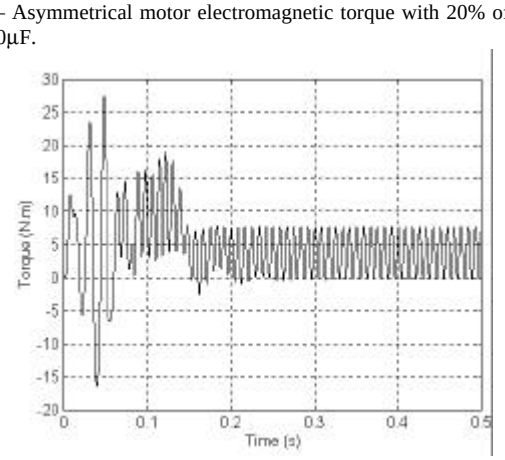

Fig. 5 – Asymmetrical motor electromagnetic torque with 50% of load – C_{ap} = 40μF.

Figures from 3 to 8 show the behavior of electromagnetic torque for different load conditions, considering only the first harmonic (fundamental), with the use of a fixed capacitor in steady state condition. In other words in those figures it is not shown the dynamic capacitance switching. It can also be observed that only at nominal condition the capacitance promotes a substantial

reduction on electromagnetic torque. In the case of loading outside the nominal condition they show large oscillations, even by keeping the same turns relationship between phases.

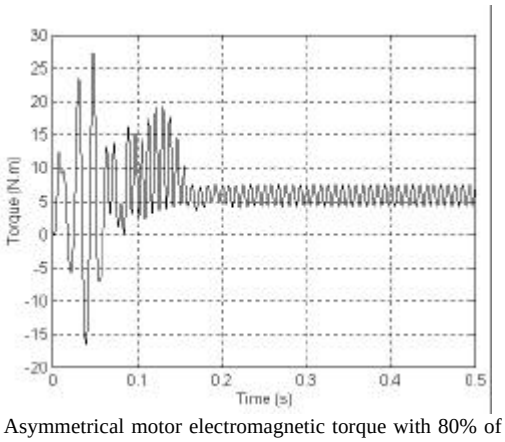

Fig. 6 – Asymmetrical motor electromagnetic torque with 80% of load – C_{ap} = 40μF.

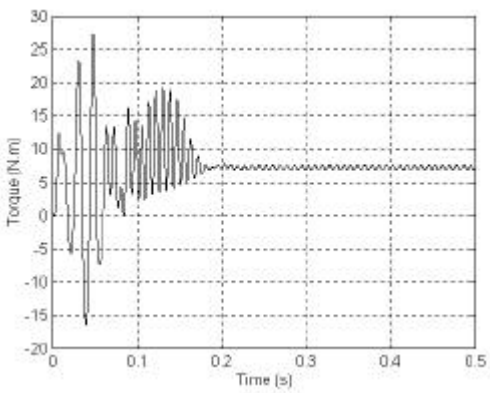

Fig. 7 – Asymmetrical motor electromagnetic torque at 100% of load – C_{ap} = 40μF.

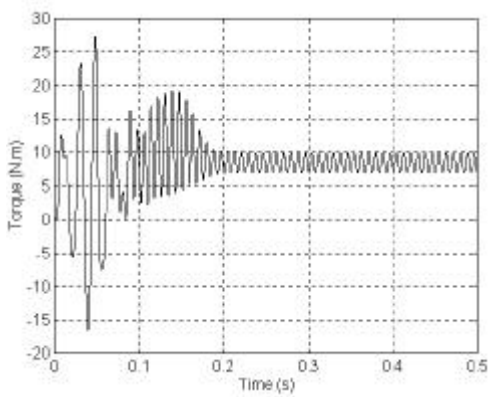

Fig 8 – Asymmetrical motor electromagnetic torque at 120% of load – C_{ap} = 40μF.

From figures 9 to 14 can be observed that keeping the same turns relationship and the same capacitor for the nominal condition, a torque oscillation reduction can be obtained for different loading from the nominal. The solution in this case is to adequate the dynamic capacitance switching for each load condition.

Figures from 9 to 14 show the behavior of electromagnetic torque for different load conditions, considering only the first harmonic (fundamental). In this case it is considered the capacitance dynamic switching along its operational range.

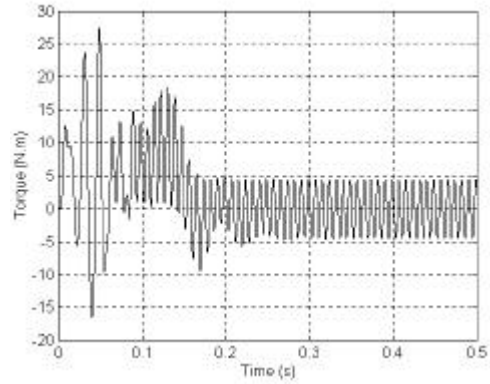

Fig. 9 – Asymmetrical induction motor electromagnetic torque at no load condition - C_{ap} = 15 μF.

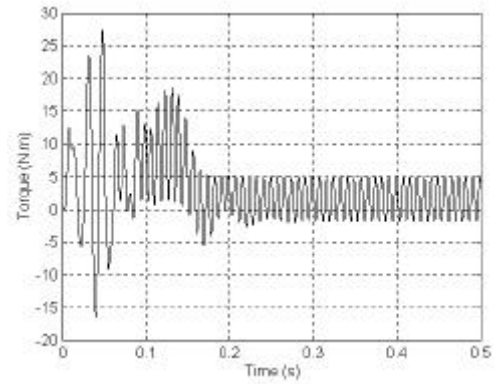

Fig. 10 – Asymmetrical induction motor electromagnetic torque at 20% of load - C_{ap} = 20 μF.

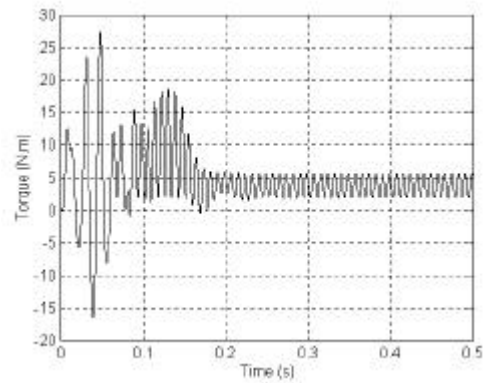

Fig. 11 – Asymmetrical induction motor electromagnetic torque at 50% of load - C_{ap} = 27 μF.

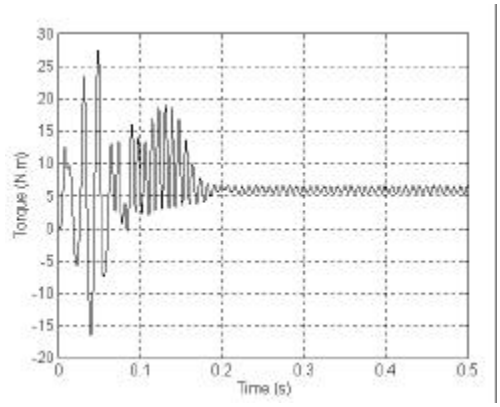

Fig. 12 – Asymmetrical induction motor electromagnetic torque at 80% of load - C_{ap} = 35 μF.

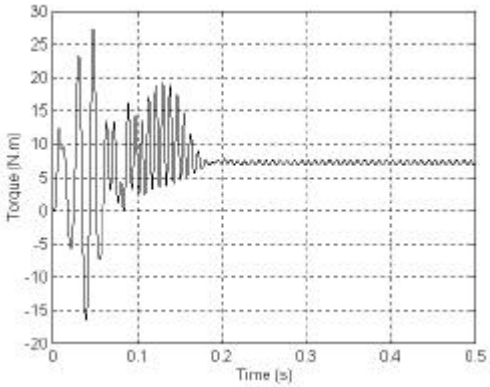

Fig. 13 – Asymmetrical induction motor electromagnetic torque at 100% of load - C_{ap} = 40 μF.

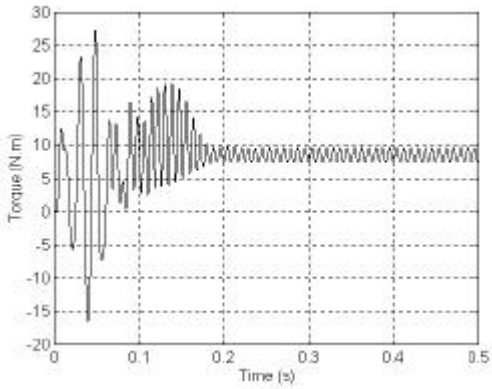

Fig. 14 – Asymmetrical induction motor electromagnetic torque at 120% of load - C_{ap} = 43 μF.

At figures 15 and 16, can be seen clearly a typical load variation for the asymmetrical three-phase induction motor. The reduction of oscillations for the motor in this case can also be seen clearly in the above mentioned figures, where it is used the dynamic capacitance switching.

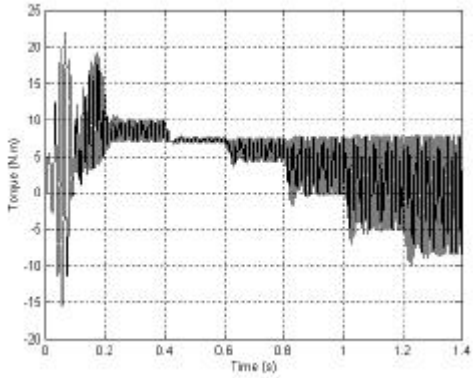

Fig. 15 – Asymmetrical motor electromagnetic torque with load variation and without the dynamic capacitance switching.

Table II below, gives the capacitor data used by the dynamic switching for each load applied to the motor.

TABLE II
LOAD VARIATION AND CAPACITANCE
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Load (%)	120	100	80	50	20	0
C_{ap} (μF)	43	40	35	27	20	15

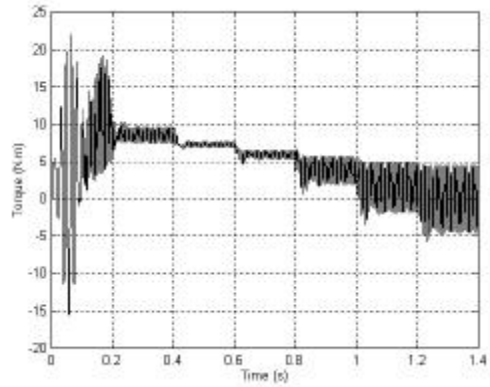

Fig. 16 – Asymmetrical motor electromagnetic torque with load variation and with the dynamic capacitance switching.

V. Conclusion

The asymmetrical three-phase induction motor with single-phase feeding, presents satisfactory behavior when the capacitance is designed for nominal condition. But with the same capacitance the motor didn't show a good electromagnetic torque performance in respect to oscillations when outside the normal load condition. This can be seen in figures 3 to 8. Therefore, with a dynamic capacitance switching with different capacitor values for each load condition, a satisfactory performance can be obtained along the motor load range, as can be seen in figures 9 to 14. It is clear then, that from figures 15 and 16, from dynamic capacitance switching, the asymmetrical three-phase induction motor with single-phase feeding shows a satisfactory performance in its whole loading range.

VI. References

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