A Repetitive Controller with Parameters Tuned *a Priori* by a Robust Controller

Cassiano Rech, Hilton Abílio Gründling, José Renes Pinheiro
UFSM/CT/DELC/NUPEDEE
CEP: 97105-900 – Santa Maria, RS – Brazil
E-mail: cassiano_rech@hotmail.com, ghilton@pequim.ctlab.ufsm.br, renes@pequim.ctlab.ufsm.br

***Abstract* – In this paper a repetitive controller with parameters tuned *a priori* by a robust controller is proposed, for uninterruptible power supply applications. The proposed control scheme can eliminate periodic distortions resulted by unknown periodic load disturbances. The implemented PWM inverter system with sinusoidal reference employs a low cost microcontroller. Most of low cost microcontrollers available in the market presents low processing speed. Due to this fact it is imperious to develop simple algorithms with few operations, so that its implementation is realizable. Principles and fundamentals, stability and special features of the control scheme are discussed. Simulation and experimental results are presented to verify the performance of the proposed approach under periodic load disturbances.**

I. Introduction

Closed-loop regulated pulsewidth modulated (PWM) inverters have been widely applied in various types of ac power conditioning systems, such as uninterruptible power supply (UPS) systems. The output voltage of the high-performance UPS system must be sinusoidal with low total harmonic distortion (THD) under both transient or periodic load disturbances. The performance of the system is measured in terms of transient response due to sudden changes at load, waveform distortion with linear and nonlinear loads, and efficiency.

Repetitive control theory [1] provides an alternative to eliminate periodic error occurred in a dynamic system. The repetitive control improves accuracy of the transient and steady-state responses of a control system when reference input signals and disturbances are periodic, consisting of the harmonic components of a common fundamental frequency. Several repetitive control schemes have been developed and applied to various industrial applications [2]-[4]. Haneyoshi et al. [5] had applied the repetitive control technique to eliminate periodic distortions that can appear in a PWM inverter. However, the error in output voltage is increased by the repetitive control if the disturbance is non-harmonic.

For eliminating this problem, Yeh and Tzou [6] presented a adaptive repetitive control scheme that employs an auxiliary compensator to stabilize the closed-loop system even with variations in the plant. Gründling et al. [7] presented a robust model reference adaptive controller including a repetitive control for UPS applications, that can effectively eliminate periodic waveform distortion resulting by unknown periodic disturbances, and it is globally stable in the presence of unmodeled dynamics.

In this paper a repetitive controller based in auxiliary states is proposed, using control parameters tuned *a priori* by a modified least-squares algorithm, which has been presented in [7] for uninterruptible power supply applications. The implementation of this algorithm is done using a low cost microcontroller, imposing the need of an algorithm with few operations.

In Section II the plant model of the system is described. The OSAP controller with a repetitive controller is described in Section III. Its performance is compared with the repetitive controller based in auxiliary states presented and discussed in Section IV. Section V presents simulation results with nonlinear loads and a discussion of the results is performed. In the Section VI is presented experimental results obtained for linear and nonlinear loads based in a microcontroller-controlled system.

II. Description of the Plant Model

Fig. 1 shows the single-phase PWM inverter, where the inverter, LC filter, and resistive load R are considered as the plant of a closed-loop digital feedback system with a sinusoidal reference. The nonlinear (triac plus resistor or rectifier plus RC filter) load causes a periodic disturbance.

The power switches are turned on and off once during each sampling interval *T*, such that V_{in} is a voltage pulse of magnitude V_B, 0 (zero) or $-V_B$ and width ΔT.

The system transfer function of Fig. 1 is given by

$$\frac{y}{u}=G(s)=\frac{\omega_n^2}{s^2+2\varsigma\omega_n s+\omega_n^2}\,, \tag{1}$$

where $\omega_n=1/\sqrt{LC}$ and $\varsigma=1/(2RC\omega_n)$, $y=V_c$ is the system output and $u=V_{in}$ is the system input.

From (1) a difference equation can be obtained using an appropriate sampling interval *T*,

$$y(k+1)=a_1y(k)+a_2y(k-1)+b_1u(k)+b_2u(k-1)+v(k+1) \tag{2}$$

where v is the measurement noise in output of the plant and $v(k)\approx(0,\sigma^2)$.

Also, (2) can be represented by,

$$y(k+1)=\phi^T(k)\theta+v(k+1)\,, \tag{3}$$

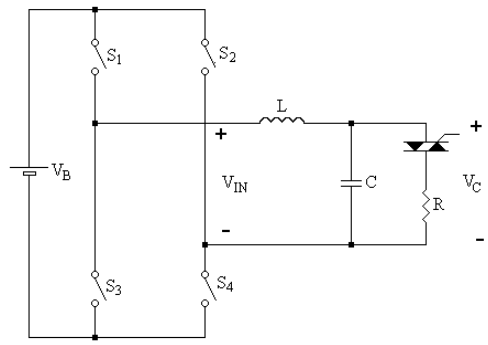


Fig. 1 – PWM inverter system.

where $\phi^T(k) = [y(k) \quad y(k-1) \quad u(k) \quad u(k-1)]$ and $\theta^T = [a_1 \quad a_2 \quad b_1 \quad b_2]$.

III. OSAP Controller with Repetitive

Consider the closed-loop control system show in Fig. 2.

It is desired to obtain a control law such that the output error is minimized and that satisfies the following quadratic performance index *J*,

$$J(k+1) = E\left[(y(k+1) - r(k+1))^2\right], \tag{4}$$

where *E* denotes mathematical expectation.

From (3) and (4) is obtained,

$$\begin{aligned} J(k+1) = E\big[&(\phi^T(k)\theta - r(k+1))^2 + \ldots \\ &\ldots + 2(\phi^T(k) - r(k+1))v(k+1) + v^2(k+1)\big]. \end{aligned} \tag{5}$$

As, for hypothesis $E[v(k+1)] = 0$ and $E[(v(k+1))^2] = \sigma^2$, then

$$J(k+1) = E[(\phi^T(k)\theta - r(k+1))^2] + \sigma^2. \tag{6}$$

Therefore, the minimal value of the performance index is obtained when

$$\phi^T(k)\theta - r(k+1) = 0. \tag{7}$$

From (7), the control law that minimizes (4) is given by

$$u_{OSAP}(k) = \frac{r(k+1) - a_1 y(k) - a_2 y(k-1) - b_2 u(k-1)}{b_1}, \tag{8}$$

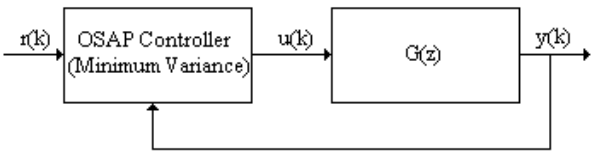


Fig 2 – Block diagram of control system with OSAP.

which is denominated OSAP (*One Sample Ahead Preview*) controller.

For the case in which the load is linear, the use of an OSAP controller (8) is sufficient to minimize the output error of the plant. This scheme does not minimize the inherent errors of the measurements and the errors of the generation of the PWM signal (V_{in}) at the input of the filter. Otherwise, for a nonlinear load varying periodically in the time, a repetitive controller is added to OSAP controller, as shown in Fig.3. This procedure minimizes the periodic disturbances due to load cyclic variation. In the same form that in [5], the equation of the repetitive controller (RP controller) is given by

$$u_{RP}(k) = c_1 e(k+1-n) + c_2 \sum_{i=1}^{\infty} e(k+1-i.n), \tag{9}$$

where $e(k) = r(k) - y(k)$, c_1 and c_2 are the gains of the repetitive controller and *n* is the number of samples in a period of output voltage. The control law $u_P(k)$ is given by

$$u_P(k) = u_{OSAP}(k) + u_{RP}(k). \tag{10}$$

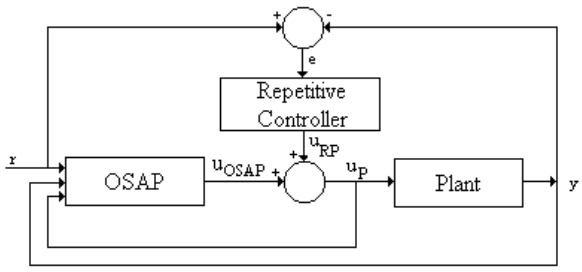


Fig 3 – Block diagram of the control system with OSAP-RP.

IV. Repetitive Controller Based in Auxiliary States

A. Structure of the Controller

Consider a single-input single-output plant as presented in Fig. 4. The input *u* and the output *y* are used to generate a *N*-1 dimensional auxiliary vectors, where *N* is the order of the system to be controlled. Such that

$$\begin{aligned} \omega_1(k+1) &= F\omega_1(k) + q.y(k) \\ \omega_2(k+1) &= F\omega_2(k) + q.u(k) \end{aligned}, \tag{11}$$

where F is a stable matrix and (*F*,*q*) is a controllable pair.

Such as it is defined in [8], (*F*,*q*) is the state space realization of $\alpha(z)/\Lambda(z)$, i.e.,

$$(zI - F)^{-1} q = \frac{\alpha(z)}{\Lambda(z)}. \tag{12}$$

The polynomial $\Lambda(z)$ is chosen as being monic, Hurwitz of degree *N*-1 and its poles must be matched with the dynamics of the model reference as well as with the dynamics of the known part (nominal model) of the plant.

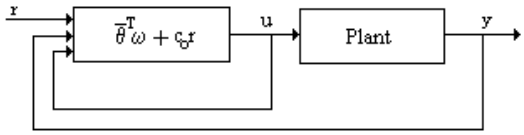


Fig. 4 – Block diagram of control system with the controller based in auxiliary states.

The control law is given by

$$u(k)=\bar{\theta}^T\omega(k)+c_o r(k)\,, \tag{13}$$

such as in [7], where $\bar{\theta}^T=[\bar{\theta}_1 \quad \bar{\theta}_2 \quad \bar{\theta}_3]$ is a (2*N*-1) dimensional control parameter vector. It is tuned *a priori* by a robust controller of the type RMRAC, using a modified least-squares adaptation algorithm. And $\omega^T=[\omega_1 \quad \omega_2 \quad y]$ is a vector containing the auxiliary states ω_1 and ω_2 and the output *y*, and c_o is a scalar feedforward parameter.

In Fig. 5, a repetitive controller is added to the controller based in auxiliary states to minimize the periodic disturbances due to periodic load variations. The control law u_P is given by

$$u_P(k)=u(k)+u_{RP}(k)\,. \tag{14}$$

B. Stability Analysis

After z-transforming of (1) and (13), the system transfer function presented in the Fig. 4 becomes

$$\left[1-\bar{\theta}_1 G(z)(z-F)^{-1}q-\bar{\theta}_2(z-F)^{-1}q-\bar{\theta}_3 G(z)\right]y=G(z)r\,, \tag{15}$$

or

$$y=G^*(z)r\,, \tag{16}$$

where

$$G^*(z)=\frac{G(z)}{1-\bar{\theta}_1 G(z)(z-F)^{-1}q-\bar{\theta}_2(z-F)^{-1}q-\bar{\theta}_3 G(z)}\,, \tag{17}$$

and *G(z)* is the z-transform of the plant transfer function.

Therefore, a necessary and sufficient condition for that the closed-loop system shown in Fig. 4 is stable, is that the closed-loop characteristic polynomial, given by

$$P_n(z)=[1-\bar{\theta}_1 G(z)(z-F)^{-1}q-\bar{\theta}_2(z-F)^{-1}q-\bar{\theta}_3 G(z)]\,, \tag{18}$$

has all its poles inside the unit circle.

With the inclusion of the repetitive control, the system transfer function which is shown in Fig. 5, after z-transforming of (1) and (14), is given by

$$\frac{E(z)}{R(z)}=\frac{(1-G^*(z))(1-z^{-n})}{1-z^{-n}H^*(z)}\,, \tag{19}$$

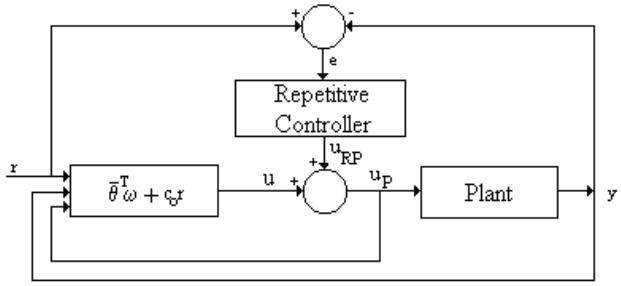


Fig. 5 – Block diagram of the proposed control system.

where the z-transform of the output error *e* is *E(z)*, *R(z)* is the transformed reference input, and

$$H^*(z)=1-z(c_1+c_2-c_1 z^{-n})G^*(z)\,. \tag{20}$$

Assuming that (17) is stable, then the stability of the system is determined by the repetitive control. From (19), a sufficient condition [5] for the stability is

$$|H^*(j\omega)|\le 1, \tag{21}$$

where ω is the angular frequency of the reference input.

V. Simulation Results

Table I lists the fundamental parameters of the single-phase PWM inverter system and reference model.

TABLE I – PARAMETERS OF PWM INVERTER AND REFERENCE MODEL

Filter inductance	L = 5,43 mH
Filter capacitance	C = 81 μF
DC input voltage	E = 40V
Reference voltage	V_{REF} = 30V, f = 60 Hz
Load resistance	R = 25 Ω
Sampling time	T = (1/2040) s

In Fig. 6 is presented the response of the OSAP-RP controller for a rated resistive load with phase commutated at angle 72º.

Fig. 7 shows the adaptation of the controller's parameters for a resistive load and a rated resistive load with phase commutated at angle 72º, using a modified least-squares adaptation algorithm. The parameters were

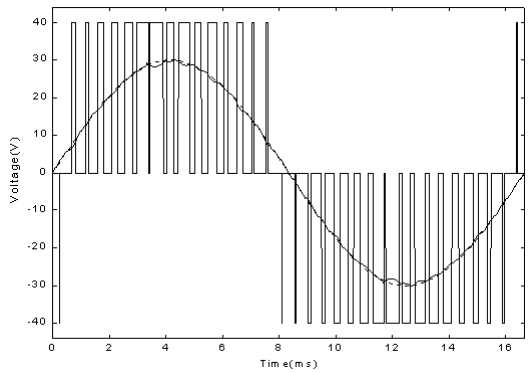
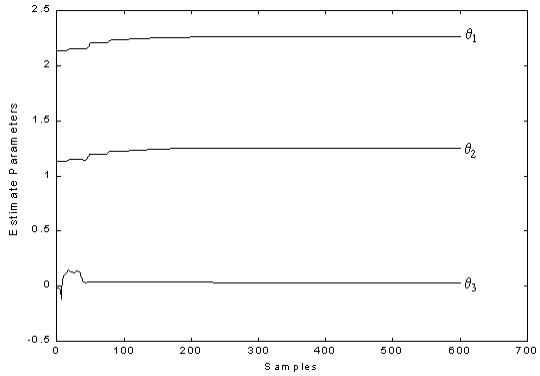
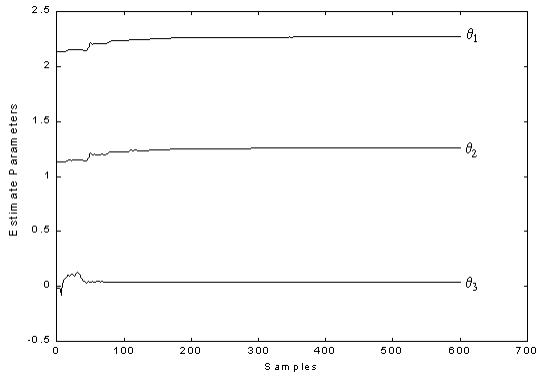


Fig. 6 – Output voltage V_C and filter input voltage V_{IN} of the OSAP-RP.



(a)



(b)

Fig. 7 – Estimate parameter using the RMRAC controller. (a) resistive load. (b) resistive load with phase commutated a angle 72º.

adapted through a robust model reference adaptive controller, using the following discrete reference model

$$W_m(z) = \frac{0.2412953.z + 0.222265}{z^2 - 1.321439.z + 0.784999}. \qquad (22)$$

Taking into account that the controller's parameters does not change significantly, as it can be seen in Fig. 7, it becomes possible to fix the parameters *a priori*. Therefore, as results, a reduced computation effort is required in the implementation.

Fig. 8 shows the response of the repetitive controller based in auxiliary states with parameters tuned *a priori*, for a rated resistive load with phase commutated at angle 72º.

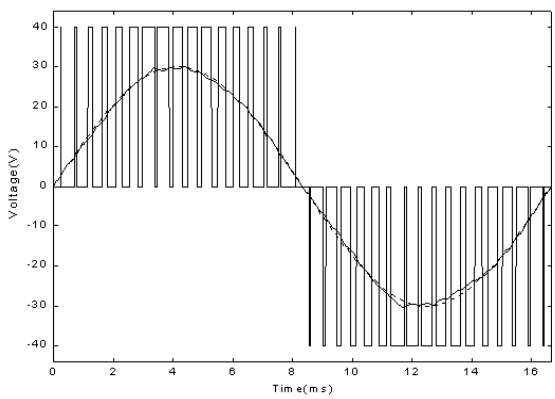


Fig. 8 - Output voltage V_C and filter input voltage V_{IN} of the repetitive controller based in auxiliary states.

VI. Experimental Results

A prototype of the PWM inverter has been built in laboratory to verify the performance of the repetitive controller based in auxiliary states with parameters tuned *a priori*. This preliminary results are taken to validate the proposed discrete controller applied to UPS. The simplified diagram of the inverter and rectifier load is shown in Fig. 9. The component values of the inverter and load are given in Table II.

Table II – Parameters of PWM inverter and load.

Filter inductance	L = 5,43 mH
Filter capacitance	C = 81 μF
DC input voltage	E = 40V
Reference voltage	V_{REF} = 30V, f = 60 Hz
Load resistance	R = 25 Ω
Load capacitance	C_L = 330 μF
Sampling interval	T = (1/2040) s

The controller has been implemented using an 8 bits microcontroller PIC17C756 of Microchip Technology Inc.. It has an embedded 10 bits A/D converter and a PWM signal generator. These features reduces significantly the circuitry required to control the PWM inverter without penalizing the cost.

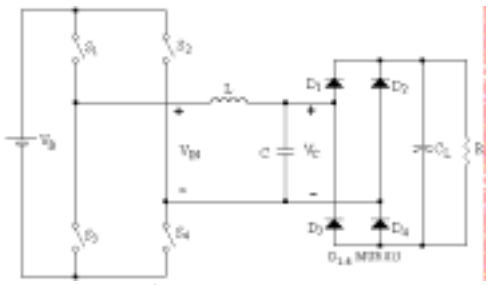


Fig. 9 – PWM inverter system.

Fig. 10 shows the waveform of the output voltage V_C for a resistive load of 25 Ω. Fig. 11 shows the output voltage V_C and input current of the rectifier and Fig. 12 shows the output voltage V_C and the input filter voltage V_{in} waveform with a load (D_{1-4}, C_L, R). By Fig. 12 can be seen a high drop of the input source voltage, allowing to observe its influence into response. In Fig. 13 is shown the FFT of the output voltage for a rectifier-RC load, where THD = 5.8 %.

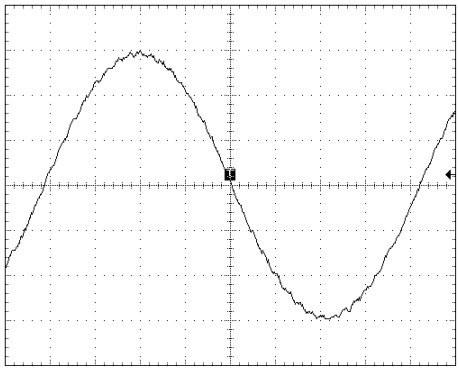


Fig. 10 - Output voltage V_C (10 V/div) with R = 25 Ω. Time scale 2 ms/div.

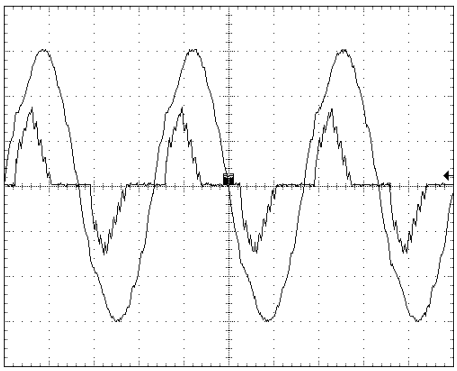


Fig. 11 – Output voltage V_C (10 V/div) and i_{LOAD} (2 A/div) with $(D_{1\text{-}4}, C_L, R)_{LOAD}$. Time scale 5 ms/div.

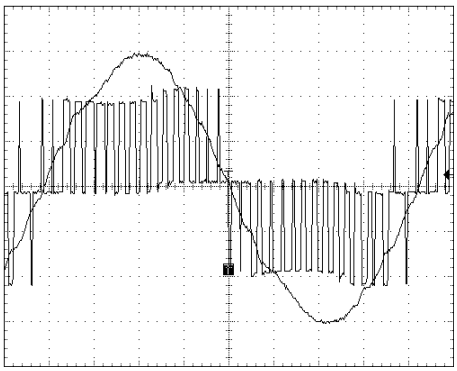


Fig. 12 – Output voltage V_C (10 V/div) and input filter voltage V_{IN} (20 V/div) with $(D_{1\text{-}4}, C_L, R)_{LOAD}$. Time scale 2 ms/div.

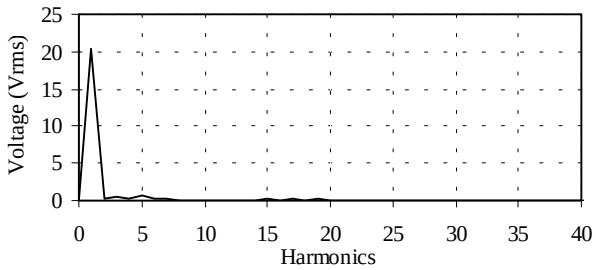


Fig. 13 – FFT of output voltage - THD = 5.8 %.

VII. Conclusions

This papers describes a repetitive controller based in auxiliary states for UPS applications. Simulation and experimental results show that the proposed control scheme can minimize the periodic waveform distortion resulting by unknown periodic disturbances. The use of the vector of parameters $\bar{\theta}^T$ tuned *a priori* by simulation based on RMRAC reduces the mathematical operations significantly. This procedure becomes possible the implementation of the algorithm in a low cost microcontroller, at low and medium switching frequencies. Even with the limitations imposed by this component, as the processing speed and the operation with integer numbers, this control algorithm presents a good response performance using the microcontroller PIC17C756. As verified by preliminary obtained results, even in the presence of the cyclic non linear loads.

VII. References

[1] S. Hara, Y. Yamamoto, T. Omata, M. Nakano, "Repetitive Control System: A New Type Servo System for Periodic Exogenous Signals", *IEEE Trans. on Automatic Control*, v. 33, no. 7, pp. 659-667, Jul. 1988.

[2] T. Inoue, "Practical Repetitive Control System Design", *Proceedings of the 29th Conference on Decision and Control*, Honolulu, Hawaii, Dec. 1990.

[3] F. Kobayashi, S. Hara, H. Tanaka, "Reduction of Motor Speed Fluctuation using Repetitive Control", *Proceedings of the 29th Conference on Decision and Control*, Honolulu, Hawaii, Dec. 1990.

[4] M. Ikeda, M. Takano, "Repetitive Control for Systems with Nonzero Relative Degree", *Proceedings of the 29th Conference on Decision and Control*, Honolulu, Hawaii, Dec. 1990.

[5] T. Haneyoshi, A. Kawamura, R. G. Hoft, "Waveform Compensation of PWM Inverter with Cyclic Fluctuating Loads", *IEEE Trans. on Industry Applications*, v. 24, n. 4, pp. 582-588, Jul./Aug. 1988.

[6] S. C. Yeh, Y. Y. Tzou, "Adaptive Repetitive Control of a PWM Inverter for AC Voltage Regulation with Low Harmonic Distortion", in *IEEE Power Electronics Specialists Conference*, pp. 157-163, 1995.

[7] H. A. Gründling, E. G. Carati, J. R. Pinheiro, "A Robust Model Reference Adaptive Controller for UPS Applications", *IEEE Ind. Electronics Conference*, v. 1, pp. 901-905, 1997.

[8] P. A. Ioannou, J. Sun, *Robust Adaptive Control*. Upper Saddle River, New Jersey, 1996, pp. 333-372.