A NOVEL ALGORITHM FOR ESTIMATING ROTOR POSITION AND SPEED IN PERMANENT MAGNET AC DRIVES

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***Abstract* - This paper presents an algorithm for speed and rotor position estimation in permanent magnet synchronous motors (PMSMs). Stator voltage and current measurements are used to build a nonlinear reduced order observer. Back Electromotive Forces (EMFs) are estimated first and then rotor position and speed are reconstructed from the estimated EMFs. The observer is implemented and tested in a permanent magnet AC drive (PMACD). The experimental results show that our observer gives a good estimation of speed and rotor position in a wide speed range.**

I. INTRODUCTION

Recently, great attention has been paid to improve the performance of electrical drives using asynchronous and synchronous motors. Advances in the technology of magnetic materials, electronic devices and VLSI circuits provide the tools for designing high performance adjustable speed drives [1]. These drives need a precise knowledge of rotor position to synchronize the stator currents with the rotor. For different reasons, in some applications it is desirable to avoid mechanical sensors. This fact has encouraged the research on the sensorless operation of Permanent Magnet AC Drives (PMACDs).

Rotor position and speed may be estimated with an observer based upon measurements of electrical variables of the motor. Different approaches to estimate the state variables can be found in the literature. In [2] and [3] nonlinear full order observers are employed for speed estimation. They suggest obtaining rotor position by integrating the speed estimate in open loop, though this is subject to biases. In [4] an algorithm to estimate flux and current by integration of differential equations was proposed. In [5] and [6] the implementation of an Extended Kalman Filter (EKF) is proposed for speed and rotor position estimation. This approach makes use of a linearization of the system model around operating points. Though the EKF is well known, the convergence of the estimated speed and position are difficult to guarantee.

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An alternative approach is to use reduced order observers to reduce the computation load. Among them, reduced order Luenberger observers can be found in their linear and nonlinear versions. The linear solution can be found in [7] and [8] while a nonlinear solution was proposed in [9]. In both types of observers the EMF is estimated first. Then, rotor position and speed are obtained from estimated EMF. The EMF must be estimated with low error in order to obtain good estimates of the rotor variables.

In this paper a novel algorithm for estimating rotor position and speed of PMSMs is presented. This algorithm estimates the back electromotive forces (EMFs) using a nonlinear reduced order observer and rotor position and speed are reconstructed from them [9]. The proposed observer presents exponential convergence of the EMFs estimation error. This is the main advantage with respect to the approximations introduced in [7] and [8]. In section II the motor model is presented. The nonlinear reduced order observer used to estimate mechanical variables is presented in section III. In section IV the algorithm behavior is tested in a commercial PMACD and experimental results are shown. In section V conclusions are drawn.

II. MOTOR MODEL

The motor is described in a stationary two-axes reference frame. Thus, the model voltages and currents are related to the actual physical quantities by the following linear transformations:

$$\begin{bmatrix} \upsilon_\alpha \\ \upsilon_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \upsilon_a \\ \upsilon_b \\ \upsilon_c \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{2}$$

where ν_α, ν_β, i_α and i_β are voltages and currents in the stationary two-axes reference frame; ν_a, ν_b , ν_c, i_a, i_b and i_c are phase voltages and phase currents. The factor 2/3 in both equations means that the transformation preserves the amplitude of voltages and currents, thus it is not power conservative.

The dynamic model for a motor with P pairs of rotor poles is given by:

$$\frac{d\theta}{dt} = \omega \tag{3}$$

$$\frac{d\omega}{dt} = \frac{K_T}{J}[-i_\alpha sin(P\theta) + i_\beta \cos(P\theta)] - \frac{B}{J}\omega \tag{4}$$

$$\frac{di_\alpha}{dt} = -\frac{R}{L} i_\alpha + \frac{K_E}{L}\omega\, sin(P\theta) + \frac{\upsilon_\alpha}{L} \tag{5}$$

$$\frac{di_\beta}{dt} = -\frac{R}{L} i_\beta - \frac{K_E}{L}\omega \cos(P\theta) + \frac{\upsilon_\beta}{L} \tag{6}$$

where the electrical parameters: R, L and K_E are resistance, inductance and EMFs constant, respectively. The mechanical variables and parameters, θ, ω, B, J, and K_T, are rotor position, rotor speed, viscosity, inertia, and torque constant, respectively.

III. Position and speed estimation via EMFs

Taking into account the motor electrical equations (5) and (6), it is clear that the rotor position and speed information is contained in the back electromotive forces (EMFs) terms. These are given by:

$$e_\alpha = -K_E\, \omega\, sin(P\theta) \tag{7}$$

$$e_\beta = K_E\, \omega\, cos(P\theta) \tag{8}$$

Taking the time derivative of (7) and (8), and replacing (7) and (8) in (5) and (6) the dynamical model of the machine can be rewritten as follows:

$$\frac{de_\alpha}{dt} = \frac{K_E K_T}{J}\left[\frac{e_\alpha^2}{e_\alpha^2 + e_\beta^2} i_\alpha + \frac{e_\alpha e_\beta}{e_\alpha^2 + e_\beta^2} i_\beta\right] - \frac{B}{J} e_\alpha - \frac{P}{K_E} e_\beta \sqrt{e_\alpha^2 + e_\beta^2} \tag{9}$$

$$\frac{de_\beta}{dt} = \frac{K_E K_T}{J}\left[\frac{e_\alpha e_\beta}{e_\alpha^2 + e_\beta^2} i_\alpha + \frac{e_\beta^2}{e_\alpha^2 + e_\beta^2} i_\beta\right] - \frac{B}{J} e_\beta + \frac{P}{K_E} e_\alpha \sqrt{e_\alpha^2 + e_\beta^2} \tag{10}$$

$$\frac{di_\alpha}{dt} = -\frac{R}{L} i_\alpha - \frac{1}{L} e_\alpha + \frac{\upsilon_\alpha}{L} \tag{11}$$

$$\frac{di_\beta}{dt} = -\frac{R}{L} i_\beta - \frac{1}{L} e_\beta + \frac{\upsilon_\beta}{L} \tag{12}$$

A nonlinear reduced order observer can be designed for estimating the EMFs (e_α, e_β). The observer copies the nonlinear dynamics (eqns. (9) and (10)) and adds correction terms such that the following equations are obtained:

$$\frac{d\hat{e}_\alpha}{dt} = \frac{K_E K_T}{J}\left[\frac{\hat{e}_\alpha^2}{\hat{e}_\alpha^2 + \hat{e}_\beta^2} i_\alpha + \frac{\hat{e}_\alpha \hat{e}_\beta}{\hat{e}_\alpha^2 + \hat{e}_\beta^2} i_\beta\right] - \frac{B}{J}\hat{e}_\alpha - \frac{P}{K_E}\hat{e}_\beta \sqrt{\hat{e}_\alpha^2 + \hat{e}_\beta^2} + L\, g\left(\frac{di_\alpha}{dt} - \frac{d\hat{i}_\alpha}{dt}\right) \tag{13}$$

$$\frac{d\hat{e}_\beta}{dt} = \frac{K_E K_T}{J}\left[\frac{\hat{e}_\alpha \hat{e}_\beta}{\hat{e}_\alpha^2 + \hat{e}_\beta^2} i_\alpha + \frac{\hat{e}_\beta^2}{\hat{e}_\alpha^2 + \hat{e}_\beta^2} i_\beta\right] - \frac{B}{J}\hat{e}_\beta + \frac{P}{K_E}\hat{e}_\alpha \sqrt{\hat{e}_\alpha^2 + \hat{e}_\beta^2} + L\, g\left(\frac{di_\beta}{dt} - \frac{d\hat{i}_\beta}{dt}\right) \tag{14}$$

where hat stands for estimated variables and

$$\frac{d\hat{i}_\alpha}{dt} = -\frac{R}{L} i_\alpha - \frac{1}{L}\hat{e}_\alpha + \frac{\upsilon_\alpha}{L}$$

$$\frac{d\hat{i}_\beta}{dt} = -\frac{R}{L} i_\beta - \frac{1}{L}\hat{e}_\beta + \frac{\upsilon_\beta}{L}$$

In this observer only the g constant is to be designed. A g value guaranteeing exponential convergence of the estimation error can be found using Lyapunov techniques as was demonstrated in [9].

Note that the correction terms use the time derivative of stator currents. In order to avoid the differentiation of the measurements, the observer equations are modified considering the following change of variables:

$$\begin{aligned} \hat{\upsilon}_1 &= \hat{e}_\alpha - L\, g\, i_\alpha \\ \hat{\upsilon}_2 &= \hat{e}_\beta - L\, g\, i_\beta \end{aligned} \tag{15}$$

In this way, the modified observer's equations are given by:

$$\frac{d\hat{\upsilon}_1}{dt} = \frac{K_E K_T}{J}\frac{\hat{e}_\alpha\,(i_\alpha \hat{e}_\alpha + i_\beta \hat{e}_\beta)}{(\hat{e}_\alpha^2 + \hat{e}_\beta^2)} - \frac{B}{J}\hat{e}_\alpha - \frac{P}{K_E}\hat{e}_\beta \sqrt{\hat{e}_\alpha^2 + \hat{e}_\beta^2} + g\,(R i_\alpha + \hat{e}_\alpha - \upsilon_\alpha) \tag{16}$$

$$\frac{d\hat{\upsilon}_2}{dt} = \frac{K_E K_T}{J}\frac{\hat{e}_\beta\,(i_\alpha \hat{e}_\alpha + i_\beta \hat{e}_\beta)}{(\hat{e}_\alpha^2 + \hat{e}_\beta^2)} - \frac{B}{J}\hat{e}_\beta - \frac{P}{K_E}\hat{e}_\alpha \sqrt{\hat{e}_\alpha^2 + \hat{e}_\beta^2} + g\,(R i_\beta + \hat{e}_\beta - \upsilon_\beta) \tag{17}$$

$$\hat{e}_\alpha = \hat{\upsilon}_1 + L\, g\, i_\alpha \tag{18}$$

$$\hat{e}_\beta = \hat{\upsilon}_2 + L\, g\, i_\beta \tag{19}$$

Then, the estimation of the rotor position and speed are calculated as follows:

$$\hat{\theta} = \frac{1}{P}\, tg^{-1}\left(-\frac{\hat{e}_\alpha}{\hat{e}_\beta} \right) \tag{20}$$

$$\hat{\omega} = \frac{1}{K_E}\sqrt{\hat{e}_\alpha^2 + \hat{e}_\beta^2} \tag{21}$$

Hence the proposed observer is represented by the second-order dynamic system given by (16) and (17), and four algebraic equations which allow to calculate the back EMFs (18) and (19), the rotor position (20) and the rotor speed (21).

IV. Experimental results

The proposed algorithm was tested with a commercial PMAC drive as shown in Fig.1. The drive consists of a PMSM and an electronic converter with a speed control loop. The control and converter forces a quasi-square wave current into the motor phases. This represents a high exigency to our observer because it is sensitive to the time derivatives of the phase currents.

The proposed algorithm runs in a PC with a Pentium of 166MHz, with a sampling period of 200μsec. Two discrete difference equations for (16) and (17) are implemented using the Euler method. The measured voltages and currents are converted to the quadrature components (1) and (2) to be inputted to the difference equations. Then the algebraic eqns. (18) trough (21) are built to obtain the rotor position and speed estimation. These estimations are converted to analog variables using 12 bits D/ACs and are visualized in a digital oscilloscope. The speed estimation is compared with the actual speed, which is provided by the tachometer of the drive; the speed error signal is obtained in the digital oscilloscope. The observer gain g is fixed at 400. This value satisfies the trade off between a fast convergence of the algorithm, and acceptable sensitiveness to current ripple and noise. If a sinusoidal PWM inverter is used to drive the motor, then it is possible to increase the value of g improving the robustness of the algorithm against mechanical parameter uncertainties.

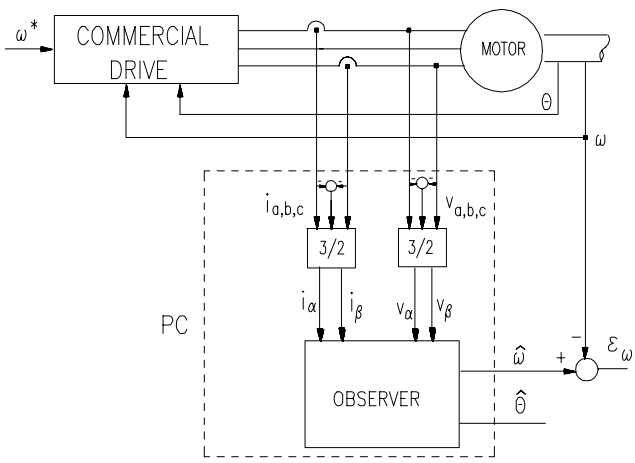

Fig.1 Block Diagram of the experimental set-up.

The data and parameters of the motor are: 0.75 kW (nominal power), 2000 rpm (nominal speed), 3 pairs of poles, 2.63 ohm (stator phase resistance), 4.5 mHy (stator phase inductance), 0.156 V/rad sec-1 (EMF constant), 0.81 Nm/A (torque constant), 4.7 A (nominal stator current), 5.7 Kgcm2 (inertia). The inverter switching frequency is 8 kHz and the DC bus voltage is 240 V. The stator voltages and currents were measured using standard Hall effect sensors. They are filtered with antialiasing filters with cut-off frequency at 500Hz, and then they are fed to 12bit A/D converters at the input of the PC equipment.

Figs 2 through 8 show the experimental results obtained with the above-described experimental set-up. For the first test we consider the motor running at medium speed (500 rpm) and we show different aspects related with the measurements of the commercial drive. Fig. 2 shows the FEMs estimations, which are the basis of the behavior of the whole observer. As expected they are two sinusoids with 90° phase shift, but they present some spikes which are due to the high value of the time derivatives of the phase currents.

The effects of these derivatives on the rotor position and speed estimations are presented in Figs. 3 and 4. Fig. 3 shows the current of one of the phases of the motor in the bottom trace. The speed estimation is shown in the upper trace. Taking into account that there is a current transition every 60°, it is evident that the ripple in the speed estimation is a consequence of the transition in the phase currents, the same as the spikes in the estimated FEMs. It is possible to diminish this ripple decreasing the gain of the observer. In this case the convergence of the observer would be slower, and its robustness against mechanical

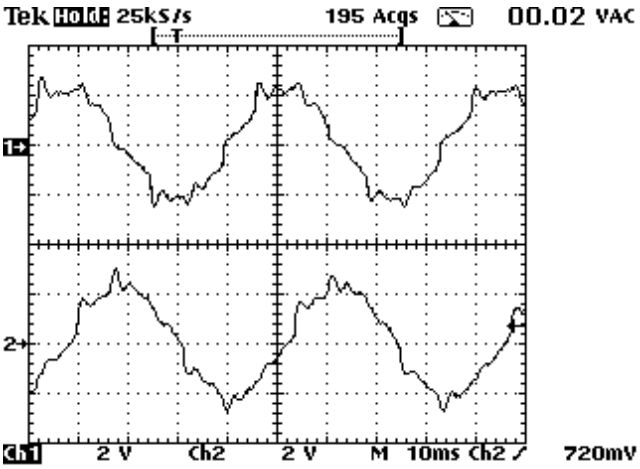

Fig. 2.: Estimation of the motor FEMs. Upper trace: e_α. Bottom trace: e_β Scales voltage: 200 V/div., time:10ms/div.

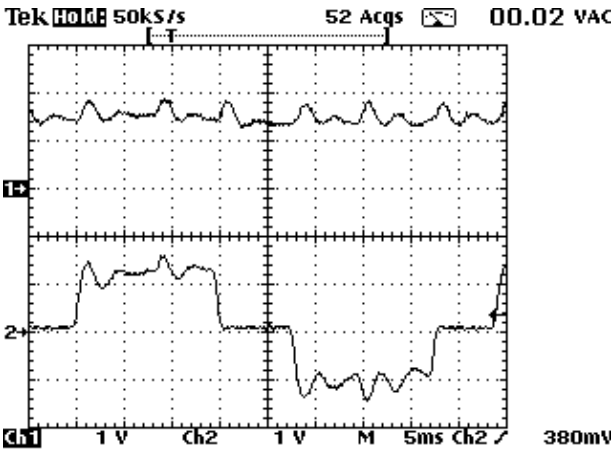

Fig. 3: Upper trace: $\hat{\omega}$: 333 rpm/div; Bottom trace: phase current: 2 A/div., t: 5ms/div.

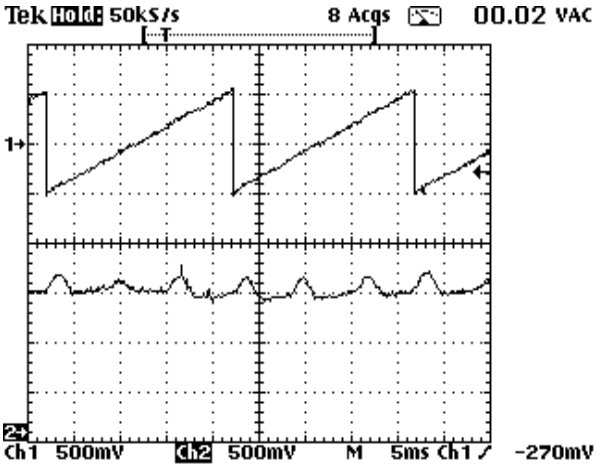

Fig.4 Upper trace: $\hat{\theta}$: π/2 rad/div; Bottom trace $\hat{\omega}$: 166 rpm/div; t: 5 ms/div.

parameter uncertainties will decrease. So we have adopted the value of 400 as a trade off, we tolerate some ripple and we preserve the robustness of the observer. Fig. 4 shows the corresponding estimations of rotor position and speed, again while the motor is running at 500 rpm. It is seen that the estimation of the rotor position is less sensitive to the current transitions. This is due to the fact the ripple in the FEMs estimations is somewhat cancelled when we take their relationship to calculate the phase. It is clear that this situation can be easily overcome using a PWM inverter, which forces sinusoidal currents into the motor.

Once the behavior of the observer and the commercial drive were determined, we examine the performance of the proposed observer in the whole speed range. Fig. 5 presents the high speed operation. The motor is running at nominal speed and the rotor speed is estimated with almost no error. The upper trace corresponds to the estimated speed, the bottom trace to the actual speed, measured from the tachometer of the drive. The scale for the estimated speed was adjusted to fit that of the tachometer (3 V corresponds to 1000 rpm). the speed error was calculated with the mathematical function of the digital oscilloscope. Fig. 6 presents the low speed operation, while the motor is

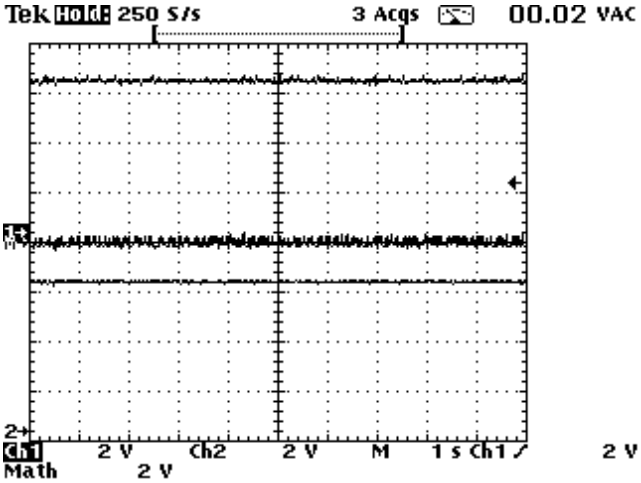

Fig 5. High speed operation Upper trace $\hat{\omega}$, mid trace $\hat{\omega} - \omega$, bottom trace ω. Scales: speed: 666 rpm/div, time: 1s/div.

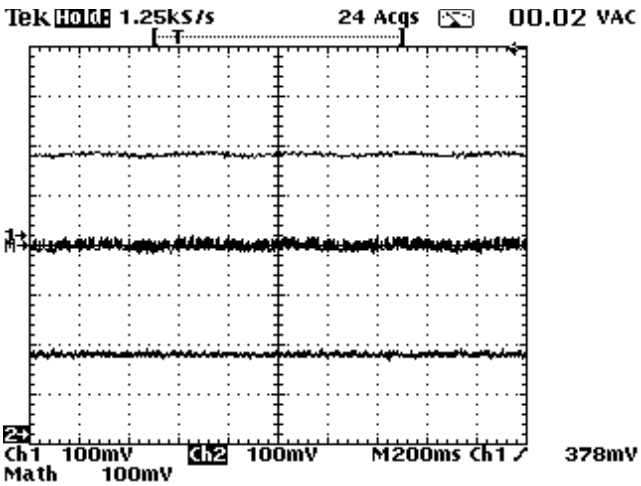

Fig 6. Low speed operation (ω=50rpm). Uppertrace $\hat{\omega}$, mid trace $\hat{\omega} - \omega$, bottom trace ω. Scales: speed: 33 rpm/div, time: 200 ms/div.

running at 50 rpm which corresponds to 1/40 of the nominal speed. The three traces are equivalent to those in Fig. 5. It is evident that the estimation at low speed is as good as that of nominal speed. So we conclude that the observer is capable of estimating the rotor speed in a very wide range.

Other test, which was carried out, consisted in the application of a trapezoidal reference speed profile. The motor is running at 100 rpm, then it is accelerated to 2000 rpm (nominal speed) and then the speed command is decreased back to 100rpm. The Fig. 7 shows the response of the estimated speed to this varying speed profile. Again the upper trace corresponds to the estimated speed, the bottom trace to the actual speed, measured from the tachometer of the drive, and the mid trace is the error calculate by the oscilloscope. It can be seen that the transient in the motor speed does not affect the quality of

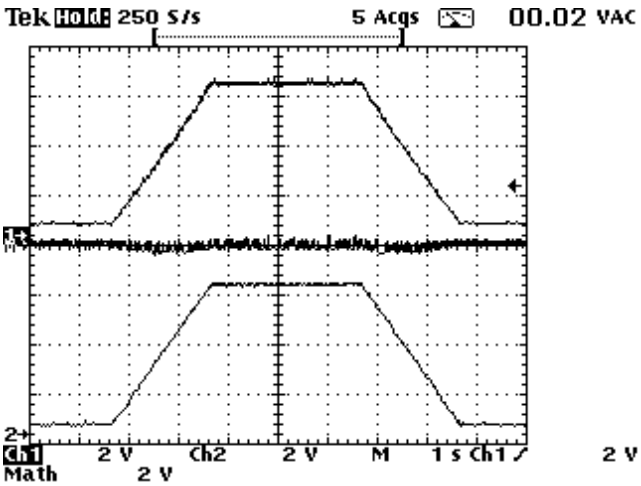

Fig. 7: Acceleration and deceleration of the drive. Uppertrace: $\hat{\omega}$, mid trace $\hat{\omega} - \omega$, bottom trace: ω. Scales: speed: 666 rpm/div, time: 1 s/div.

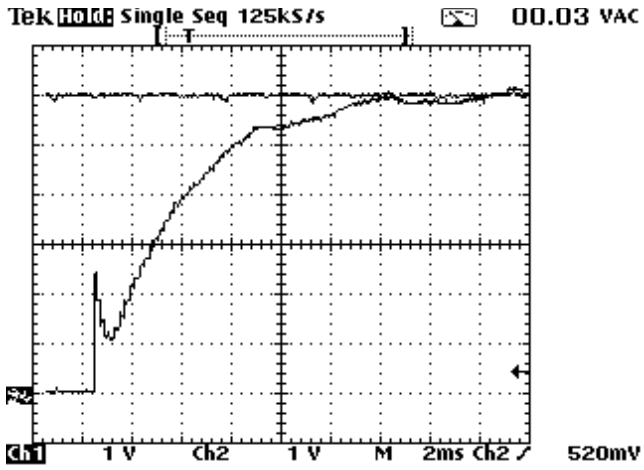

Fig 8. Convergence of the proposed observer. Scales: $\hat{\omega}$. 666 rpm/div, t:2ms/div.

the speed estimation. This demonstrates that the proposed observer is capable of estimating constant and varying speed.

Finally, in Fig. 9 we show the speed of convergence of the proposed observer. In this test, the motor is running at nominal speed, and the observer is started with an initial estimation of 700 rpm and 0 radians. The straight line in the figure corresponds to the measurement of the actual speed. The other trace is initially in zero, since the observer is no functioning. When the observer is started the speed estimation takes the initial value and after a transient it converges to the actual speed in about 16 msec. This quick convergence allows implementing faster controllers of the drive without any risk of interactions of both dynamics.

V. Conclusions

In this paper a novel algorithm for estimating rotor position and speed was presented. A nonlinear reduced order observer is used for estimating the back electromotive forces. Then, mechanical variables are reconstructed from those estimates. The main feature of the proposed algorithm is the exponential convergence of EMFs estimation error. As a consequence the rotor position and speed estimates converge to the true values in exponential way. The algorithm was implemented in a PC environment and it was tested measuring the voltages and currents of a commercial PMAC drive. The experimental results shown a very satisfactory performance of the proposed observer over a wide speed range.

VI. References

[1] Novotny D. W. and T. A. Lipo, *Vector Control and Dynamics of AC Drives*, Clarendon Press, Oxford UK, 1996.

[2] Jones L. and J. Lang, “A State Observer for the Permanent-Magnet Synchronous Motor”, *IEEE Trans. on Ind. Elec.*, vol.36, pp. 374-382, Aug. 1989.

[3] Low T., T. Lee and K. Chang, “A Nonlinear Speed Observer for Permanent-Magnet Synchronous Motor”, *IEEE Trans. on Ind. Elec.*, vol. 40 pp. 307-315, June 1993.

[4] Ertugrul N. and P. Acarnley, "A New Algorithm for Sensorless Operation of Permanent Magnet Motors", *IEEE Trans. on Ind. Appl.*, Vol. 30, pp. 126-133, 1994.

[5] Dhaouadi R., N. Mohan and L. Norum, "Design and Implementation of an Extended Kalman Filter for the State Estimation of a Permanent Magnet Synchronous Motor", *IEEE Trans. on Power Electronic*, Vol. 6, pp. 491-497, 1991.

[6] Bolognani S., M. Zigliotto, M. Zordan, “Rotor position detection for sensorless PM synchronous motor drives”, *Power Electronics and Motion Control Conference (PEMC'98)*, Prague, Check Republic, pp. 8-83-8-88, 1998.

[7] Orlowska-Kowalska T., "Comparison of Speed Estimation Methods for the Sensorless Permanent Magnet AC Drives", *Power Electronics and Motion Control Conference (PEMC'98)*, Prague, Czech Republic, pp. 5-266-5-271, September 1998.

[8] Tomita M., T. Senjyu, S. Doki and S. Okuma, "New Sensorless Control for Brushless DC Motors Using Disturbance Observers and Adaptive Velocity Estimations", *IEEE Trans. on Ind. Elect.*,Vol. 45, pp.274-282, 1998.

[9] Solsona J., M.I. Valla and C. Muravchik, "A Nonlinear Reduced Order Observer for Permanent Magnet Synchronous Motors", *IEEE Trans. on Ind. Elect.*, Vol. 43, pp.492-497, 1996.