Instantaneous Evaluation of Power Quality Indexes

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***Abstract* – This article describes two time domain decompositions, which can be applied in asymmetric conditions and non-sinusoidal waveforms, making possible to evaluate in real time several power quality indexes. A digital monitoring system capable of monitoring simultaneously active and non-active powers, displacement and power factors, harmonic distortion, flicker level and unbalancing factors, is also presented. Experimental results show the viability of the proposed methodology.**

I. INTRODUCTION

Worldwide electrical system modifications and the increasing market globalization, produced the necessity to certify the quality of the supplied power.

Voltage and current disturbing indexes, such as total harmonic distortion (THD), flicker level, sequence components, sags and swells became of major importance to identify anomalies and to quantify the disturbing effects in the power system. The recent instantaneous power theories can also provide valuable contributions to this power quality monitoring process.

Many authors [5-8] have presented different methods to calculate the electric power, both in time or in frequency domain. The main problem is that very detailed theories usually are not simple to apply in practical conditions. Normally this is due to technology limitations.

This paper describes a methodology to obtain several power quantities in the electrical network, associating them with the physical phenomena, which originate the disturbances.

II. TIME DECOMPOSITION APPROACH

The proposed methodology is based on two main decompositions, which provide the extraction of desired quantities and indexes. Both decompositions are realized in time domain.

A. Decomposition into Fundamental and Residual Signals

The first decomposition is applied to instantaneous voltage and current signals. The basic idea is to separate the input signals into fundamental waves (x_1) and residual signals (x_{res}). This last one is responsible for power quality deterioration and is directly related with the high frequency components.

$$x_{in}(t) = x_1(t) + x_{res}(t) \tag{1}$$

where:
$x_{in}(t)$ - *input voltage or current signals;*
$x_1(t)$ - *fundamental wave;*
$x_{res}(t)$ - *residual signal.*

This separation is obtained using a digital notch filter tuned at the presumed fundamental frequency. The residual signal of the output of this filter is then recombined with the input, thus providing the fundamental wave without phase shift. Details of this process are described in [1].

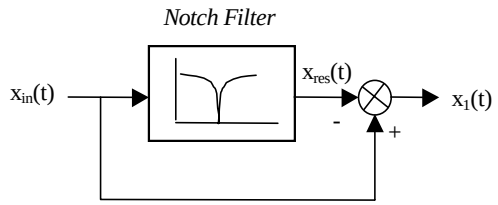

Figure 1 – Fundamental and residual detection.

In order that the notch filter works adequately, it is necessary that the network fundamental frequency variations are negligible. Nevertheless, adaptive filtering techniques can provide the tracking of slow changes of the frequency in the range ±3 Hz.

B. Decomposition into Active and Non-Active Currents

The second important decomposition is applied only to the input currents and provides the splitting for in-phase and orthogonal components [6,8]. These components have been named *instantaneous active current* and *instantaneous non-active current*, respectively. This decomposition is very convenient to define power indexes relating the different kinds of electric powers present in a system.

The above mentioned active/non-active decomposition is based on the *Lagrange Multipliers Method*, which is applicable to any functions irrespective of the variables (phases) number or analysis domain.

In multi-phase systems the instantaneous active power is expressed by the scalar product (•) between instantaneous voltage and current vectors, such as:

$$p(t) = [v_n(t)]^T \bullet [i_n(t)] \tag{2}$$

where:
$[v_n(t)]^T$ - *transpose of voltage functions vector;*
n - *phases [a,b,c...] of the multi-phase system.*

Applying the *Lagrange´s Method* to the scalar function defined in (2), the *instantaneous active currents* are obtained by the following transformation:

$$[i_{p_n}(t)] = \frac{p(t)}{\|v_n(t)\|^2}.[v_n(t)] \tag{3}$$

where:

$||v_n(t)||$ - *Euclidian norm of voltages* $[v_n(t)]$;

$p(t)/||v_n(t)||^2$ - *instantaneous Lagrange multiplier function;*

$[i_{pn}(t)]$ - *instantaneous active currents (in phase with voltages* $[v_n(t)]$*).*

The *instantaneous non-active currents* are obtained as the difference between the input currents and the active currents:

$$[i_{q_n}(t)] = [i_n(t)] - [i_{p_n}(t)] \tag{4}$$

where:

$[i_n(t)]$ - *instantaneous input currents;*

$[i_{qn}(t)]$ - *instantaneous non-active currents (orthogonal with voltages* $[v_n(t)]$*).*

Applying (3) and (4) at (1), it is possible to extract the fundamental in-phase and orthogonal currents relative to the fundamental voltages, and residual (harmonic) in-phase and orthogonal currents relative to the residual (harmonic) voltages. These calculations can all be implemented in a digital algorithm.

The decompositions (*A* and *B*) may be combined in order to provide the development of useful time domain power quality indexes, which will be detailed next.

III. VOLTAGE AND CURRENT QUALITY INDEXES

Applying the described decompositions to a three-phase system, the following voltage and current quality indexes will be evaluated as time functions:

- Total harmonic distortion of voltages and currents (THD_v , THD_i);
- Sequence components of voltages and currents (V_+ , V_- V_0 , I_+ , I_- , I_0);
- Instantaneous flicker sensation (S_f).

A. *Total Harmonic Distortion of Voltages and Currents*

The total harmonic distortion (THD) of a signal is usually defined as the square root of a number of normalized harmonics amplitudes:

$$THD_x = \frac{\sqrt{\sum_{h=2}^{50} X_h^2}}{X_1} = \sqrt{\sum_{h=2}^{50}\left(\frac{X_h}{X_1}\right)^2} \tag{5}$$

However, with the separation of the fundamental wave from the disturbing input signal (1), the total harmonic distortion can be estimated by a RMS calculation algorithm using a moving window corresponding to one fundamental period of the following time function:

$$THD_x = \sqrt{\frac{1}{T}\int^{T}\left(\frac{x_{res}(t)}{x_1(t)}\right)^2.dt} \tag{6}$$

where:

$x_1(t)$ - *fundamental wave of voltage or current;*

$x_{res}(t)$ - *residual signal containing harmonic components;*

T - *period of the fundamental wave.*

The proposed method (6) evaluates the harmonic distortion in time domain instead of finding the frequency content of the input signals using the classical Fast Fourier Transform. Besides getting THD as a time function, this approach also presents the advantage of including automatically the inter-harmonic contribution.

B. *Sequence Components of the Fundamental Waves*

Since the separation mechanism (1) preserves the amplitudes and phase angles of the fundamental, the *Clarke's instantaneous components* ($\alpha,\beta,0$) can be calculated using the invariant power transform:

$$\begin{bmatrix} x_\alpha(t) \\ x_\beta(t) \\ x_0(t) \end{bmatrix} = \sqrt{\frac{2}{3}}.\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.\begin{bmatrix} x_{a1}(t) \\ x_{b1}(t) \\ x_{c1}(t) \end{bmatrix} \tag{7}$$

where:

x_α, x_β, x_0 - *Clarke's instantaneous voltages or currents components;*

x_{a1}, x_{b1}, x_{c1} - *fundamental waves of a,b,c voltages or currents.*

Using an orthogonal summation of Clarke´s components, the *Fortescue's instantaneous sequence components* (+,-) are obtained as:

$$x_+(t) = \frac{1}{\sqrt{2}}[x_\alpha(t) + jx_\beta(t)] \tag{8}$$

$$x_-(t) = \frac{1}{\sqrt{2}}[x_\alpha(t) - jx_\beta(t)] \tag{9}$$

where: $\pm j$ *represents a* $\pm\frac{T}{4}$ *time shifting operation.*

The zero sequence component is obtained directly from Clarke's transformation.

Using the RMS calculation algorithm, the sequence components (X_+ , X_- , X_0) are calculated, in order to obtain the *unbalancing factors*:

$$\frac{X_0}{X_+} \quad \text{and} \quad \frac{X_-}{X_+} \tag{10}$$

C. Instantaneous Flicker Sensation

The instantaneous flicker sensation is obtained by processing the input voltages, which are demodulated and filtered according to [2]. The flicker level S_f is defined as the square mean of the frequency weighted voltage variations $\Delta v_f(t)$:

$$S_f = \frac{1}{\tau}\int_t^{t+\tau}\left(a_f.\Delta v_f(t)\right)^2.dt \qquad (11)$$

where:

τ - *300ms is the typical eye-brain time constant.*

a_f - *visual sensitivity weighting factor*

D. Considerations

The previously defined indexes (x_1, x_{res} , *THD*, X_0/X_+ X_-/X_+ *and* S_f) constitute time functions, which can be monitored using a single equipment by processing three-phase voltages and currents. Thus, the need of different instruments to evaluate each quality index is avoided.

IV. POWER DEFINITIONS

Since the voltage and current signals are available, the instantaneous electric power can also be calculated and several associated indexes can be evaluated.

The decompositions presented in (1), (3) and (4) provide the calculation of different power terms circulating in the electrical network, which can be classified as active or non-active powers, due to fundamental waves or residual (high frequency) signals.

A. Instantaneous Active and Non-Active Powers

The classical expression of the instantaneous three-phase power is:

$$p_{3\phi}(t) = v_a(t).i_a(t) + v_b(t).i_b(t) + v_c(t).i_c(t), \qquad (12)$$

where: *a,b,c represent the system's phases.*

Multiplying the input voltages by the corresponding active currents (3), gives the *instantaneous active power* :

$$p(t) = v_a(t).i_{ap}(t) + v_b(t).i_{bp}(t) + v_c(t).i_{cp}(t), \qquad (13)$$

For the fundamental waves (1) and (3) results the *instantaneous fundamental active power*:

$$p_1(t) = v_{a1}(t)i_{a1p}(t) + v_{b1}(t)i_{b1p}(t) + v_{c1}(t)i_{c1p}(t) \qquad (14)$$

In order to identify and define the difference between the total instantaneous active power and the instantaneous fundamental active power, the *instantaneous residual active power* is defined:

$$p_{res}(t) = p(t) - p_1(t) \qquad (15)$$

The product of the voltages and non-active currents Euclidian norms provides the magnitude of the *instantaneous non-active power* :

$$|q(t)| = \|v_n(t)\| \cdot \|i_{qn}(t)\|, \qquad (16)$$

where *n* represents the phase number of a multi-phase system (three-phase in this case).

The same product when applied on the fundamental waves produces the magnitude of the *instantaneous fundamental non-active power* (also named instantaneous reactive power).

$$|q_1(t)| = \|v_{n1}(t)\| \cdot \|i_{qn1}(t)\| \qquad (17)$$

The previous expressions (16 and 17) show the instantaneous information about the magnitude of the non-active power components. Their signs can be evaluated according to [8]. For compensation proposes the non-active power signs are not needed, since the control loop is usually based on instantaneous non-active currents, which contain the correct phase information.

If the signs are known, then the difference between the total instantaneous non-active power and the instantaneous fundamental non-active power, gives the *instantaneous residual non-active power*:

$$q_{res}(t) = q(t) - q_1(t) \qquad (18)$$

Now, different concepts for apparent power can be defined from active and non-active instantaneous powers, such as : the *instantaneous apparent power*

$$s = \sqrt{p^2 + q^2}, \qquad (19)$$

the *instantaneous fundamental apparent power*

$$s_1 = \sqrt{p_1^2 + q_1^2}, \qquad (20)$$

and the *instantaneous residual apparent power*

$$s_{res} = \sqrt{p_{res}^2 + q_{res}^2}. \qquad (21)$$

The above expressions can be related by the following diagram:

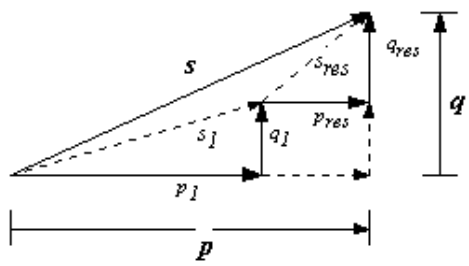

Figure 2 – Instantaneous power quadrature components.

B. Instantaneous Power Components and the Related Physical Phenomena

For the purpose of associating the previous power components with their physical interpretations, the next diagram is presented:

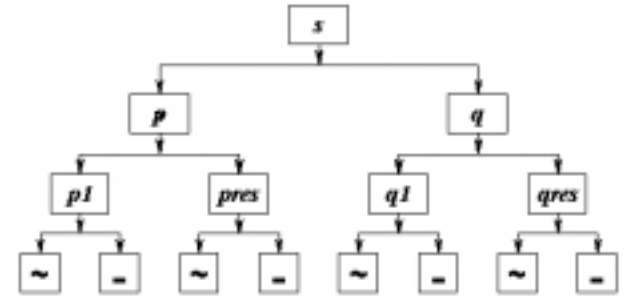

Figure 3 – Power decomposition diagram.

The final components may be interpreted as follows :

$\bar{p}_1$ and $\bar{q}_1$ are the instantaneous active and non-active power components, resulting from symmetric fundamental voltages and currents, in other words, only fundamental positive sequence component. Mean values.

$\tilde{p}_1$ and $\tilde{q}_1$ are the instantaneous active and non-active power components, resulting from asymmetric fundamental voltages and currents, in other words, fundamental negative sequence component. Oscillatory values.

$\bar{p}_{res}$ and $\bar{q}_{res}$ are the instantaneous active and non-active power components, resulting from symmetric harmonic voltages and currents with the same frequency, in other words, harmonic positive sequence components. Mean values.

$\tilde{p}_{res}$ and $\tilde{q}_{res}$ are the instantaneous active and non-active power components, resulting from asymmetric harmonic voltages and currents (harmonic negative sequence component), adding to the power component for voltages and currents with different frequencies (cross product of frequencies). Oscillatory values.

The main observation is originated from the instantaneous harmonic active power. Voltages and currents with same frequency generate average values of active power, which can be absorbed or supplied continuously by the load, depending on the situation.

V. INSTANTANEOUS POWER QUALITY INDEXES

With the foregoing instantaneous power definitions and calculations, some instantaneous power quality indexes can be derived, which carry the necessary information about electrical energy deterioration.

The usual displacement factor concept ($cos\phi_1$), or the power factor under sinusoidal situations, can be expressed instantaneously by:

$$df = cos\,\phi_1 = \frac{p_1}{s_1} \tag{22}$$

The relation of the instantaneous total active power to the instantaneous total apparent power, which circulates in the supply-consumer system, is defined as the *instantaneous power factor*:

$$pf = \frac{p}{s} = \frac{p_1 + p_{res}}{s} \tag{23}$$

An *instantaneous active power efficiency* can be defined to indicate how much active power circulating in the system realize useful work:

$$\eta_p = \frac{p_1}{p_1 + p_{res}} \tag{24}$$

From (23) and (24) the *instantaneous fundamental active power factor* can be defined *:*

$$pf_1 = \eta_p \times pf = \frac{p_1}{s}, \tag{25}$$

indicating how much of all instantaneous apparent power in the system, is converted into useful work.

VI. INSTRUMENT DESCRIPTION

Other important proposal of this article is to show the possibility of monitoring all mentioned quantities and indexes by the same digital instrument.

The monitoring system, named *Instantaneous Power Quality Monitor,* is constituted of a data acquisition interface with 8 independent channels multiplexed and a 16 bits A/D converter communicating with a PC through the parallel port.

Continuous on-line calculations require fast processing and detection of harmonic distortion demands sufficiently high sampling frequency. These are the two most conflicting problems to be solved by the monitoring system. These restrictions determined the minimum computer processing capacity (clock of 233 MHz) as well as a minimum sampling frequency for the 8 signal channels (19.2 kHz).

Processing continuously the instantaneous three-phase voltage and current samples, using Turbo C and Assembly routines, the system stores only the final results concerning the desired indexes and disturbing events, thus requiring little data storing memory.

The actual version has a graphic interface, on which the main results can be observed immediately.

VII. EXPERIMENTAL RESULTS

The following results were obtained from a programmable signal generator, which provided the following signals.

The experiment was divided in three time intervals: During the first interval ($t<t_1$), balanced three-phase voltages supplied an inductive (φ = *-30º*) load with *10%* less current in "phase c". No harmonic components were presented.

During the second interval ($t_1<t<t_2$) the unbalancing current and the phase displacement were annulled, but *3%*, *5%* and *7%* of *3rd*, *5th* and *7th* harmonic components were added to "*a,b,c*" voltage and current phases, respectively.

During the third interval ($t>t_2$), the phase angle (φ = *-30º*) between currents and voltages was introduced again, on the fundamental and the harmonic components. Fig. 4 shows the voltages and currents during the three time intervals.

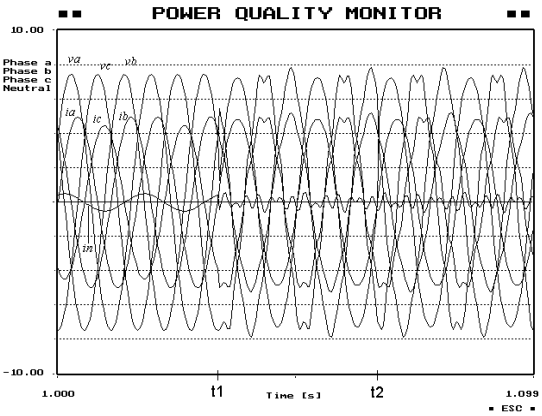

Figure 4 – Voltage and current input signals.

Fig. 4 shows the influence of asymmetries ($t<t_1$) and harmonics (t>t1) on the voltage and current signals. During the second interval, currents and voltages are clearly in-phase.

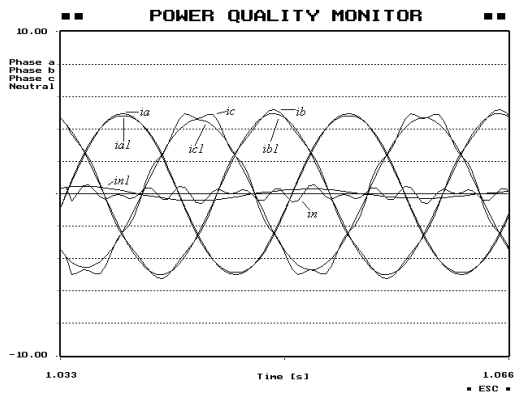

Figure 5 – Input and fundamental current signals ($t>t_1$).

Fig. 5 shows the fundamental currents corresponding to the input signals during the second interval, obtained according to decomposition (1).

Fig. 6 shows the gradual extinction of the unbalancing currents (after t_1). This effects is due to the dynamic response of the notch filter.

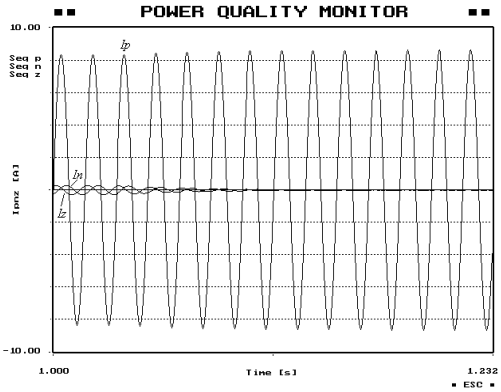

Figure 6 – Current sequence components overall evaluation.

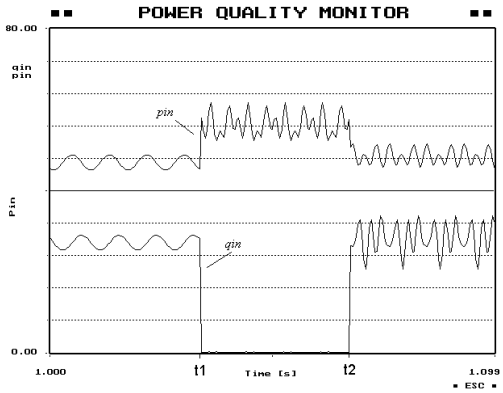

Figure 7 – Instantaneous active and non-active power during the three intervals.

In the first interval of Fig. 7, the average values of *p* and *q* are related with the fundamental power and the displacement angle, while the oscillating part is related with the current asymmetry. In the second interval, this figure confirms that only active power circulates, and the oscillatory part is due to the harmonics. In the last interval, the non-active power, corresponding to the first interval, appears again but now affected by the harmonic content.

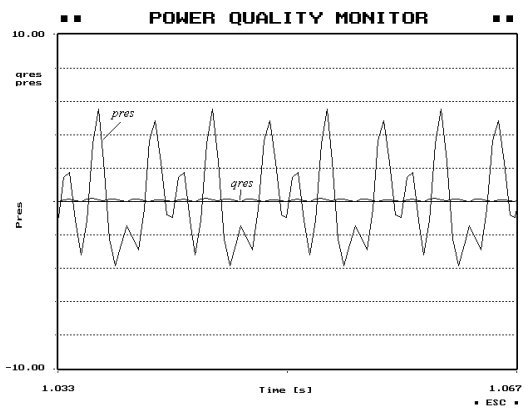

Figure 8 – Instantaneous active and non-active residual powers under conditions of interval 2.

Fig. 8 shows that during the second interval, the harmonic components contribute only by the active residual power. The mean value of p_{res} can be evaluated and thus distinguished from the total active power.

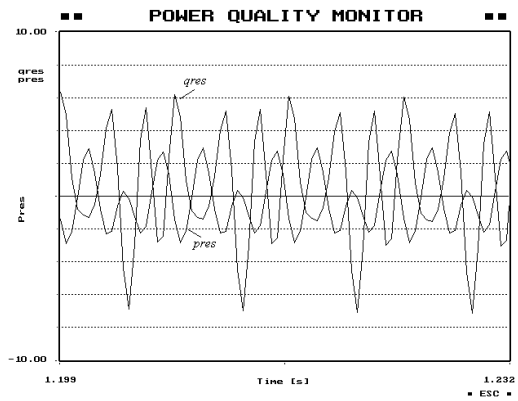

Figure 9 – Instantaneous active and non-active residual powers under conditions of interval 3.

Fig. 9 illustrates the *residual active and non-active powers*, which can contain mean and oscillatory values. The oscillatory values are related with unbalancing or in this case, cross products of voltages and currents with different (*3rd*, *5th* and *7th)* harmonic frequencies.

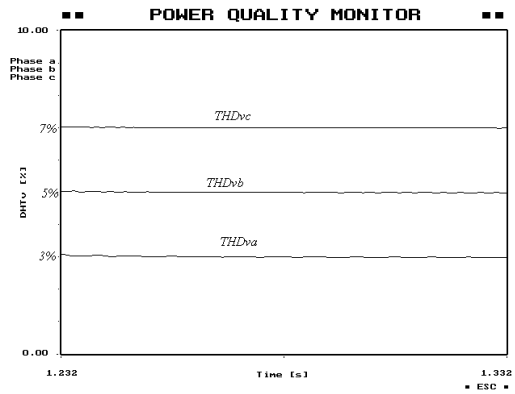

Figure 10 – Total harmonic distortion of input voltages.

Fig. 10 shows the THD in "*a,b,c*" voltages. As expected they represent exactly the 3%, 5% and 7% imposed on the voltages and currents by the generator.

VIII. CONCLUSIONS

The reasoning of the proposed time decomposition into fundamental and residual components (1) may be stated as follows:

1- the ideal power system should present only the fixed, balanced fundamental voltages and in-phase currents;
2- the disturbing effects should be identified as fast as possible regarding to their basic characteristics: waveform distortion, amplitude fluctuation or unbalancing effects;
3- since those disturbing characteristics can be evaluated continuously as time functions, quality indexes can be associated with each of them;
4- each quality index calculation constitutes by itself a very efficient data compressing algorithm.

The further decomposition of the current signals into active and non-active component associated with the decomposition into fundamental and residual, provides directly to identify and evaluate total harmonic distortion, unbalancing factors, flicker levels and several instantaneous power terms, such as:

- instantaneous active and non-active input powers;
- instantaneous active and non-active fundamental powers;
- instantaneous active and non-active residual powers.

The explicit separation of higher frequency and fundamental power terms has practical interests both for monitoring systems and compensation purposes.

According to the example:

- The average *residual active power* is generated by harmonic voltages and currents with common frequency and sequence. This harmonic power component is normally small, in the range of *0.2-0.5%* of p_1 , but should be considered.
- The *non-active residual power* component is generated either by cross products of voltages and currents with non-common frequencies or by harmonic displacement angles of common frequencies.
- The *non-active fundamental power* component is originated by the fundamental voltage and current displacement angles and can be associated with the conventional reactive power Q_1.

In this way it is possible to identify the origin of the disturbing effects and share responsibilities to extinguish or reduce the problem to acceptable levels, which is the main objective of power quality monitoring.

IX. REFERENCES

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