A NOVEL DYNAMIC MODEL FOR LINEAR INDUCTION MOTORS

Ezio Fernandes da Silva(Eng.)[1], Euler Bueno dos Santos(Dr.)[2] and Marco A. Assfalk de Oliveira(Dr.)[2]

[1]Centro Federal de Educação Tecnológica de Goiás - CEFET - GO
Rua 75 nº 46, Centro
74055-110 – Goiânia – Goiás - Brazil
efs@cefetgo.br

[2]Universidade Federal de Goiás
Escola de Engenharia Elétrica, s/n, Setor Universitário
74605-220 – Goiânia – Goiás - Brazil
ebs@eee.ufg.br

Abstract **– This paper presents a new mathematical model that describes the dynamic behaviour of a linear induction motor, divided into two portions. The first part represents the motor's dynamics without the end effects while the second portion describes the attenuation caused by the end effects on the linear induction motor.**

Keywords: **linear induction motor, thrust, linkage flux, mathematical model.**

I. INTRODUCTION

This paper describes a dynamic model of the linear induction motor (LIM), based on the dq model of the equivalent electrical circuit, taking into account the end effects. A velocity inverse function is used to express the effects caused by the LIM's velocity on the magnetization factor of the equivalent circuit [1].

Due to the complexity of the linear induction motor's electromagnetic field theory, an analysis of the LIM's equivalent circuit was undertaken, with alterations to the magnetizing inductance and a resistance representing the core losses.

In a LIM with a short primary and infinite linor (secondary), the primary is continuously entering a new linoric (secondary) region. This new linoric region tends to resist the abrupt increase of the magnetization flux penetration by permitting a gradual accumulation of the magnetizing field density in the gap. The new linori region and its influence on the magnetic field change the LIM's performance in comparison with the conventional induction motor [2].

To obtain the LIM's equivalent circuit incorporating the end effects, one must quantify these effects during entry and exit from the linor, in respect to the primary.

In the case of no relative movement between the primary and the linor, the LIM's equivalent circuit is equal to the conventional induction motor, since the contribution of the end effects will be relatively small.

Assuming that the primary is moving, an analysis of the entry end of the linor, under a uniform magnetic field, reveals that eddy currents will appear in the linor, having the effect of reducing the density of the magnetic field at the entry end of the primary. The linor current is consequently reduced exponentially over time through the linoric time constant.

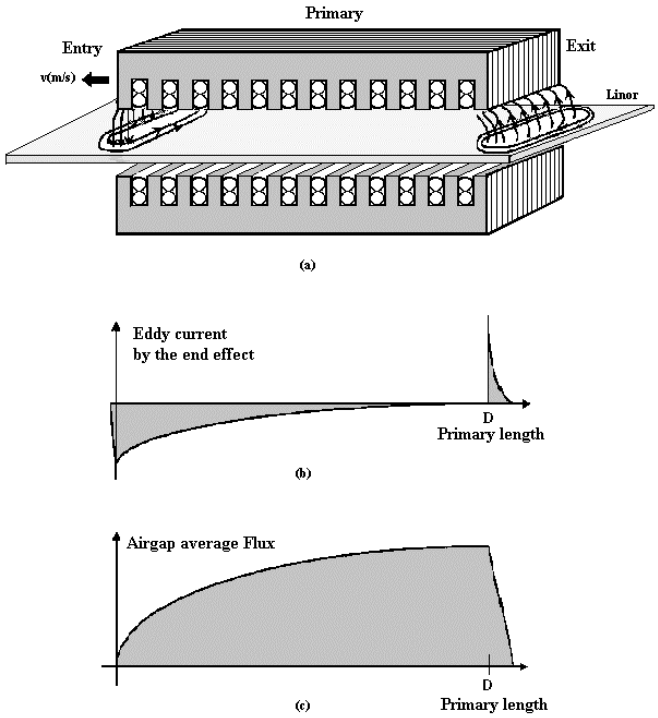

Figure 1. (a) Linor currents at the entry and exit ends for a given velocity. (b) Linor current values and polarity due to end effects. (c) airgap average magnetic flux.

Since the moving primary passes through the linoric region, the new magnetic field penetrating the linor decreases at the entry end, and increases at the exit end, i.e., the linor currents at the entry end increase very rapidly, mirroring the primary's current, and compensating for the magnetic field caused by the primary, with the effect of zeroing the magnetic flux at the entry end. On the other hand, at the exit end, the linor currents drop very rapidly, with a very short time constant, with respect to the leakage inductance of the linor.

These transient changes at the entry and exit ends as a function of the linoric time constant, are shown in figure 1.

The spatial distribution of the magnetic flux density along the primary's length depends on the relative velocity between the primary and the linor. For a zero relative velocity, the LIM's primary can be seen as infinite, in which case the end effects may be ignored.

VARIABLES

v_{ds}	primary d-axis voltage.
v_{qs}	primary q-axis voltage.
v_{dr}	linor (secondary) d-axis voltage.
v_{qr}	linor q-axis voltage.
i_{ds}	primary d-axis current.
i_{qs}	primary q-axis current.
i_{dr}	linor d-axis current.
i_{qr}	linor q-axis current.
λ_{ds}	primary d-axis linkage flux.
λ_{qs}	primary q-axis linkage flux.
λ_{dr}	linor d-axis linkage flux.
λ_{qr}	linor q-axis linkage flux.
L_s	self inductance of primary per phase.
L_r	self inductance of linor per phase.
L_{ls}	primary leakage inductance per phase.
L_{lr}	linor leakage inductance per phase.
L_m	magnetizing inductance
R_s	resistance of primary pef phase.
R_r	resistance of linor pef phase.
P	Number of poles.
D	primary length.
Q	Factor linked to the primary's length.
F	electromagnetic thrust.
v	primary velocity (m/s).
τ_p	pole pitch.
ω_s	primary electrical frequency.
ω_{sl}	slip frequency.

II. MATHEMATICAL MODEL

The end effects are not very noticeable in conventional induction motors. On the other hand, in linear induction motors, these effects become increasingly relevant with the increase in the relative velocity between the primary and the linor. The reason for that is the nonsymmetrical structure of the LIM. Therefore, the end effects will be analysed as a function of the LIM's velocity.

A. Resistance and inductance in the magnetization branch

The end effects behave differently at the entry and exit ends of the LIM. The currents induced in the linor at the entry end decay more slowly than at the exit end, due to its larger time constant [1][2].

The Q factor is associated with the length of the primary, and to a certain degree, quantifies the end effects as a function of the velocity v as described by equation (1).

$$Q = \frac{DR_r}{(L_m + L_{lr})v} \tag{1}$$

Note that the primary's length is inversely dependent on the velocity, i.e., for a zero velocity the primary's length may be considered infinite, and the end effects may be ignored, there is no difference in the equivalent circuit of LIM and conventional induction motor. As the velocity increases, the primary's length decreases, increasing the end effects, which causes a reduction of the LIM's magnetization current. This effect may be quantified in terms of the magnetization inductance with the equation:

$$L'_m = L_m(1 - f(Q)) \tag{2}$$

where

$$f(Q) = \frac{1 - e^{-Q}}{Q} \tag{3}$$

The resistance in series with the inductance (L'_m) in the magnetization branch of the equivalent circuit of the d-axis, is determined in relation to the increase in losses occurring with the increase of the currents induced at the entry and exit ends of the linor. These power losses can be represented as a serially connected linor resistor $R_r f(Q)$ in magnetizing current branch[1][2].

B. The LIM's Equivalent dq Circuit

To obtain a suitable LIM equivalent circuit, it will be necessary to quantify the effects of the entry and exit of new material on the air gap flux distribuition know as the end effect.

The linear induction motor's equivalent dq circuit is different from that of a conventional induction motor. The LIM's equivalent circuit is shown in figure 2.

The linor entry current on the q-axis maintains a null reference q-axis linor flux, within a narrow error band.

The equivalent q-axis electrical circuit for the LIM is identical to that of a conventional induction motor. In this case, the parameters are not changed by the end effects.

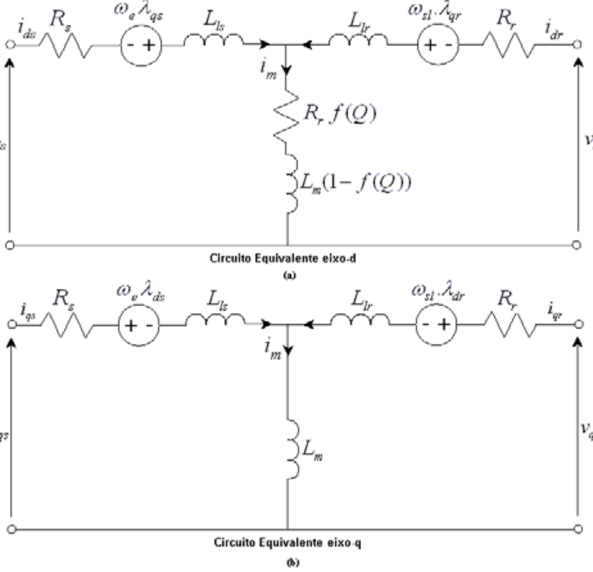

Figure 2 – The LIM equivalent circuit taking into account end effects.

The airgap flux is influenced by the linor d-axis entry currents. Therefore the equivalent d-axis electrical circuit associated with a conventional induction motor can not be used in the analysis of a linear induction motor, if the end effects, as a function of velocity, are to be considered.

B1. LIM Linor Voltage and Primary dq Voltage

The primary and linor voltage equations, using a synchronous reference system are given in [1]

$$v_{ds} = R_s i_{ds} + R_r f(Q)\left(i_{ds} + i_{dr}\right) + \frac{d\lambda_{ds}}{dt} - \omega_e \lambda_{qs} \tag{4}$$

$$v_{qs} = R_s i_{qs} + \frac{d\lambda_{qs}}{dt} + \omega_e \lambda_{ds} \tag{5}$$

$$v_{dr} = R_r i_{dr} + R_r f(Q)\left(i_{ds} + i_{dr}\right) + \frac{d\lambda_{dr}}{dt} - \omega_{sl} \lambda_{qr} \tag{6}$$

$$v_{qr} = R_r i_{qr} + \frac{d\lambda_{qr}}{dt} + \omega_{sl} \lambda_{dr} \tag{7}$$

B2. The LIM primary and linor linkage fluxes

The linkage fluxes are given by the following equations:

$$\lambda_{ds} = L_{ls} i_{ds} + L'_m \left(i_{ds} + i_{dr}\right) \tag{8}$$

$$\lambda_{qs} = L_{ls} i_{qs} + L_m \left(i_{qs} + i_{qr}\right) \tag{9}$$

$$\lambda_{dr} = L_{lr} i_{dr} + L'_m \left(i_{ds} + i_{dr}\right) \tag{10}$$

$$\lambda_{qr} = L_{lr} i_{qr} + L_m \left(i_{qs} + i_{qr}\right) \tag{11}$$

B3. LIM thrust

The thrust force is given by:

$$F_e = \frac{3\pi}{2\tau_p} \frac{P}{2} \left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}\right) \tag{12}$$

C. LIM primary and linor dq Currents

The d-axis and q-axis currents in the LIM's primary and linor are derived from equations (8) and (10).

Therefore, isolating i_{ds} from (8) and i_{dr} from (10), results in:

$$i_{ds} = \frac{\lambda_{ds} - L'_m i_{dr}}{L_{lr} + L'_m} \tag{13}$$

$$i_{dr} = \frac{\lambda_{dr} - L'_m i_{ds}}{L_{lr} + L'_m} \tag{14}$$

Isolating i_{ds} from (9) and i_{dr} from (11), we have:

$$i_{qs} = \frac{\lambda_{qs} - L_m i_{qr}}{L_s} \tag{15}$$

$$i_{qr} = \frac{\lambda_{qr} - L_m i_{qs}}{L_r} \tag{16}$$

Substitution of (14) into (13) results in:

$$i_{ds} = \frac{\left(L_r - L_m f(Q)\right)\lambda_{ds} - L_m\left(1 - f(Q)\right)\lambda_{dr}}{\left(L_s - L_m f(Q)\right)\left(L_r - L_m f(Q)\right) - L_m^2\left(1 - f(Q)\right)^2} \tag{17}$$

Substitution of (13) into (14) gives:

$$i_{dr} = \frac{\left(L_s - L_m f(Q)\right)\lambda_{ds} - L_m\left(1 - f(Q)\right)\lambda_{ds}}{\left(L_s - L_m f(Q)\right)\left(L_r - L_m f(Q)\right) - L_m^2\left(1 - f(Q)\right)^2} \tag{18}$$

Substituting (16) into (15) results in:

$$i_{qs} = \frac{L_r \lambda_{qs} - L_m \lambda_{qr}}{L_\sigma} \tag{19}$$

Substituting (15) into (16) gives:

$$i_{qr} = \frac{L_s \lambda_{qs} - L_m \lambda_{qs}}{L_\sigma} \tag{20}$$

where:

$$L_\sigma = L_s L_r - L_m^2 \tag{21}$$

D. Separating the LIM primary and linor dq Currents

The LIM's primary and linor dq currents are separable into two portions, of which the first is independent of the end effects, and the second dependent of the end effects. With this approach, the first portion behaves as a conventional induction motor current and the second portion as a attenuation function due to the LIM's end effects.

To derive these currents, note that the linkage fluxes are also separated into two parts: the first is independent of the end effects and will be indicated by the index "1", while the second describe the linkage flux dependent on the end effects, indicated by the index "2". These fluxes are given by the following equations:

$$\lambda_{ds} = \lambda_{ds1} + \lambda_{ds2} \tag{22}$$

$$\lambda_{qs} = \lambda_{qs1} + \lambda_{qs2} \tag{22}$$

$$\lambda_{dr} = \lambda_{dr1} + \lambda_{dr2} \tag{24}$$

$$\lambda_{qr} = \lambda_{qr1} + \lambda_{qr2} \tag{25}$$

Inserting equations (22) through (25) into (17) through (20), and after some mathematical operations, the following equations emerge:

$$i_{ds} = i_{ds1} + i_{ds2} \tag{26}$$

$$i_{qs} = i_{qs1} + i_{qs2} \tag{27}$$

$$i_{dr} = i_{dr1} + i_{dr2} \tag{28}$$

$$i_{qr} = i_{qr1} + i_{qr2} \tag{29}$$

where:

$$i_{ds1} = \frac{L_r \lambda_{ds1} - L_m \lambda_{dr1}}{L_\sigma} \tag{30}$$

$$i_{dr1} = \frac{L_s.\lambda_{dr1} - L_m.\lambda_{ds1}}{L_\sigma} \tag{31}$$

$$i_{ds2} = A_1 + A_2 + A_3 \tag{32}$$

$$A_1 = \frac{L_r.\lambda_{ds2} - L_m \lambda_{dr2}}{L_\sigma} \tag{33}$$

$$A_2 = \frac{L_m L_l f(Q)\left(L_r \lambda_{ds} - L_m \lambda_{dr}\right)}{L_\sigma \left(L_\sigma - L_m L_l f(Q)\right)} \tag{34}$$

$$A_3 = \frac{L_m L_\sigma f(Q)(\lambda_{ds} - \lambda_{dr})}{L_\sigma \left(L_\sigma - L_m L_l f(Q)\right)} \tag{35}$$

$$i_{dr2} = A_4 + A_5 + A_6 \tag{36}$$

$$A_4 = \frac{L_s \lambda_{dr2} - L_m \lambda_{ds2}}{L_\sigma} \tag{37}$$

$$A_5 = \frac{L_m L_l f(Q)\left(L_s \lambda_{dr} - L_m \lambda_{ds}\right)}{L_\sigma \left(L_\sigma - L_m L_l f(Q)\right)} \tag{38}$$

$$A_6 = \frac{L_m L_\sigma f(Q)(\lambda_{dr} - \lambda_{ds})}{L_\sigma \left(L_\sigma - L_m L_l f(Q)\right)} \tag{39}$$

$$i_{qs1} = \frac{L_r \lambda_{qs1} - L_m \lambda_{qr1}}{L_\sigma} \tag{40}$$

$$i_{qs2} = \frac{L_r \lambda_{qs2} - L_m \lambda_{qr2}}{L_\sigma} \tag{41}$$

$$i_{qr1} = \frac{L_s \lambda_{q1s} - L_m \lambda_{qs1}}{L_\sigma} \tag{42}$$

$$i_{qr2} = \frac{L_s \lambda_{q2s} - L_m \lambda_{qs2}}{L_\sigma} \tag{43}$$

$$L_l = L_{ls} + L_{lr} \tag{44}$$

E. Separating the linkage flux

The linkage flux are described by the following equations:

$$\frac{d\lambda_{ds1}}{dt} = v_{ds} - R_s i_{ds1} + \omega_e \lambda_{qs1} \tag{45}$$

$$\frac{d\lambda_{ds2}}{dt} = -R_s i_{ds2} + \omega_e \lambda_{qs2} - R_r f(Q)\left(i_{ds} + i_{dr}\right) \tag{46}$$

$$\frac{d\lambda_{qs1}}{dt} = v_{qs} - R_s i_{qs1} - \omega_e \lambda_{ds1} \tag{47}$$

$$\frac{d\lambda_{qs2}}{dt} = -R_s i_{qs2} - \omega_e \lambda_{ds2} \tag{48}$$

$$\frac{d\lambda_{dr1}}{dt} = -R_r i_{dr1} + \omega_{sl} \lambda_{qr1} \tag{49}$$

$$\frac{d\lambda_{dr2}}{dt} = -R_r i_{dr2} + \omega_{sl} \lambda_{qr2} - R_r f(Q)\left(i_{ds} + i_{dr}\right) \tag{50}$$

$$\frac{d\lambda_{qr1}}{dt} = -R_r i_{qr1} - \omega_{sl} \lambda_{dr1} \tag{51}$$

$$\frac{d\lambda_{qr2}}{dt} = -R_r i_{qr2} - \omega_{sl} \lambda_{dr2} \tag{52}$$

F. Separating the Thrust

The LIM's thrust force may be separated into two components, where the first is similar in expression to the conventional induction motor's thrust force and will be indicated by the index "1", while the second portion represents the attenuation caused by the end effect and will be denoted by the index "2". Therefore, the LIM thrust force may be represented by:

$$F_e = F_{e1} + F_{e2} \tag{53}$$

The component that is similar to the thrust force of a conventional induction motor is given by :

$$F_{e1} = \frac{3\pi}{2\tau_p}\frac{P}{2}\left(\lambda_{ds1} i_{qs1} - \lambda_{qs1} i_{ds1}\right) \tag{54}$$

The attenuation thrust force which arises from the end effect is given by:

$$F_{e2} = F_{e2a} - F_{e2b} \tag{55}$$

where:

$$F_{e2a} = \frac{3\pi}{2\tau_p}\frac{P}{2}\left(\lambda_{ds1} i_{qs2} + \lambda_{ds2} i_{qs1} + \lambda_{ds2} i_{qs2}\right) \tag{56}$$

$$F_{e2b} = \frac{3\pi}{2\tau_p}\frac{P}{2}\left(\lambda_{qs1} i_{ds2} - \lambda_{qs2} i_{ds1} - \lambda_{qs2} i_{ds2}\right) \tag{57}$$

III. LIM SIMULATION

The LIM parameters used in the dynamics simulation are given in table I.

TABLE I
Linear Induction Motor Parameters Used

MIL –1	
Parameters	**Values (Units)**
Primary length - D	210 mm
Primary Width	45 mm
Number of Poles - P	2
Pole pitch	98.5 mm
Linor (Secondary) thickness	4.5 mm
Number of slots	12
Gap length	8 mm
Primary resistance- R_s	5.348 Ω
Linor (secondary) resistance - R_r	11.603 Ω
Primary Inductance - L_s	0.1073 mH
Linor (secondary) inductance - L_r	0.094618 mH
Magnetizing inductance - L_s	0.09213 mH
Inertial Moment - J	0.00247 Kgm^2

The SIMULINK® dynamics model used for our simulation of the vectorial control of the LIM is shown in

figure 3.
Figure 4 shows the LIM thrust force characteristics, with the free acceleration as a function of slippage, considering a synchronous velocity of 1.95 m/s and a sychronous reference system.

In figures 5 to 7, the reference system has changed, as follows: in the [0,1] second interval, the reference is synchronous, in the (1,2] second interval, the reference is half the synchronous velocity, and between 2 and 3 seconds, the reference is stationary. Note that in the above figures, the thrust forces F_e e F_{e2} have oscillations due to the end effects, in the [1,3] second interval. In figure 7, the thrust force F_{e1} does not present oscillations because it is independent of the end effect. The thrust force (F_e and F_{e2}) oscillations caused by the end effect are not noticeable when a synchronous reference system is used.

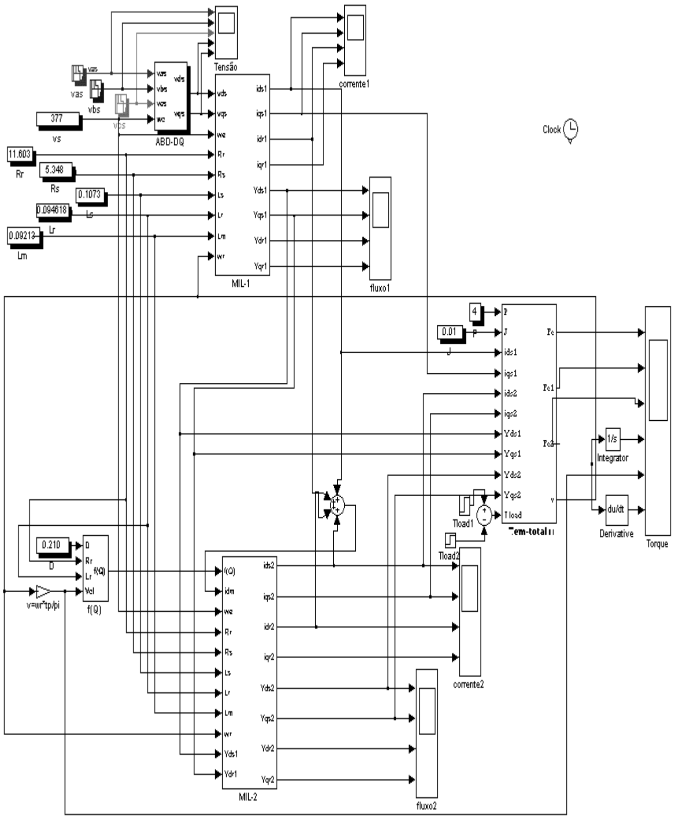

Figure 3 – LIM dynamics model, simulated using SIMULINK®.

Figure 8 contains a graph of the thrust forces F_e, F_{e1} e F_{e2} over time, using a synchronous reference system. A load of 5.5 N was applied from t=1.5 s to t=2.15 s. Note that the thrust force F_{e1} becomes greater than F_e due to compensation required for the thrust force F_{e2}, caused by the end effects.

IV. CONCLUSIONS

We have shown that the dynamics model of the linear induction motor (LIM) can be separated into end-effect-dependent and independent parts, which can be interpreted as representation of the dynamics model of the conventional induction motor (end effects ignored) plus the attenuation due to a thrust force caused by the end effects.

The thrust force oscillations present in the LIM are caused by the end effects, as can be seen in figure 6. The thrust force behaviour, without the contribution of the end effects, does not present oscillations, as shown in figure 7.

The research described here is being extended by an investigation of vectorial control methods that take into account the influences upon the LIM of end effects.

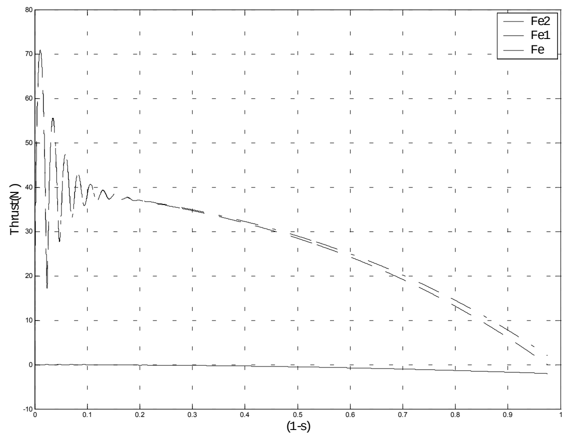

Figure 4 – Thrust forces F_e (dashed line), F_{e1} (dash-dotted line) and F_{e2} (solid line) as a function of slippage, in a synchronous reference system.

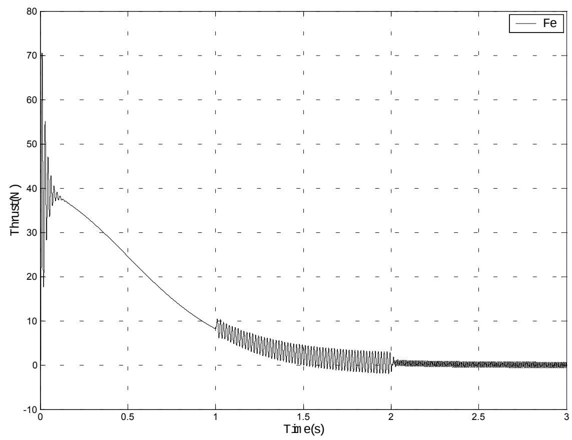

Figure 5 – Thrust force F_e over time for, respectively, a synchronous ([0,1] seconds), half synchronous ((1,2] seconds) and stationary ((2,3] seconds) reference system.

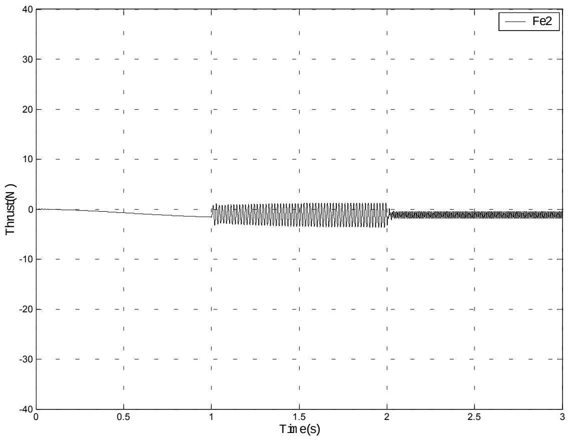

Figure 6 – Thrust force F_{e2} over time for, respectively, a synchronous ([0,1] seconds), half synchronous ((1,2] seconds) and stationary ((2,3] seconds) reference system.

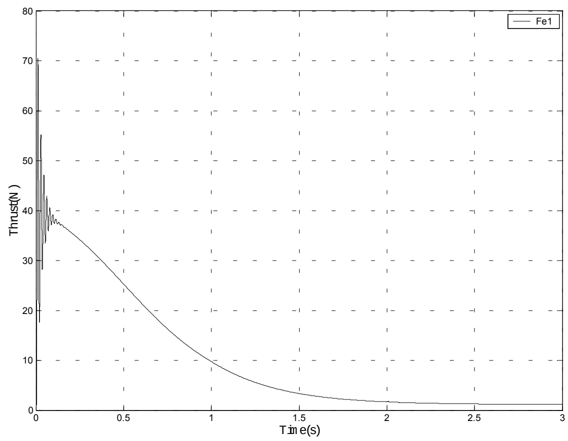

Figura 7 – Thrust force F_{e1} over time for, respectively, a synchronous ([0,1] seconds), half synchronous ((1,2] seconds) and stationary ((2,3] seconds) reference system.

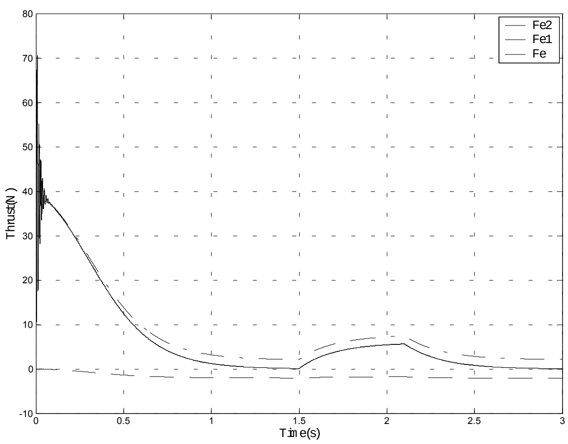

Figure 8 – Thrust forces F_e (solid line), F_{e1} (dash-dotted line) and F_{e2} (dashed line) over time for a synchronous reference system, with a load force of 5.5 N applied during the the interval between 1.5 and 2.15 seconds.

REFERENCES

[1] K.Nam, J. H Sung, “A New Approach to Vector Control for Linear Induction Motor Considering End Effects” IEEE IAS annual meeting, 3-7 Oct., in Phoenix, Arizona, pp. 2284-2289, 1999.

[2] J. Duncan, “Linear Induction Motor – Equivalent Circuit Model”. IEE Proc., Vol. 130, pt. B, nº 1, pp. 51-57, January, 1983.

[3] E.B. Santos, J.R. Camacho, A.A. Paula, “The Linear Induction Motor (LIM) Power Factor, Efficiency and Finite Element Considerations.”, ICEM 2002, CD-ROM.

[4] L.M. Neto, E.B. Santos, “Métodos para Determinação de Indutâncias de um Motor de Indução Linear”, Annals of the Congr. Brasil. de Automática - Uberlândia-MG, v. 1, pp. 231-236, 2000.

[5] J.F. Gieras, Linear Induction Drives, University of Tokyo Press, Japan, 1994.

[6] R.C. Creppe, “Uma contribuição à modelagem de Máquinas de Indução Lineares”, Tese de Doutorado, UNICAMP, Campinas, São Paulo, Brasil, 1997.

[7] D.Dolinar, G. Stumberger, G. Bojan, “Calculation of the Linear Induction Motor Model Parameters Using Finite Element” IEEE Transaction on Magnetics, Vol. 34, n. 5, pp. 3640-3643, Sep. 1998.