AN ALGORITHM FOR IMPROVING THE TRANSIENT RESPONSE OF REPETITIVE CONTROLLED PWM INVERTERS UNDER NON-PERIODIC DISTURBANCES

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Abstract – **Although repetitive controlled voltage-source PWM inverters present an excellent steady-state performance even under severe nonlinear cyclic loads, they do not present a good dynamic performance under non-periodic disturbances. Aiming to solve this problem, this paper proposes an algorithm for improving the transient response of repetitive controlled voltage-source PWM inverters under non-periodic disturbances, such as a sudden linear load change or removal of a nonlinear cyclic load. The proposed algorithm is based on the analysis of the output error behavior to identify the occurrence of a non-periodic disturbance. Once identified the occurrence of this disturbance, it is possible to reset the repetitive control action so that the repetitive controller can storage the correct information concerning this new load. Simulation and experimental results are presented (1 kVA @ 110 V_{RMS}) to verify the good performance of the proposed algorithm under different load conditions.**

I. INTRODUCTION

Control systems are typically evaluated from their steady-state performance and transient response under different input signals and/or disturbances. Consequently, several instantaneous feedback control techniques have been developed for distinct applications to achieve zero steady-state error and fast transient response.

The internal model principle [1] establishes that the output of a closed-loop system tracks an input reference signal without steady-state error if the model that generates this reference is included in the stable feedback system. For instance, it is well known that the transfer function $1/s$ must be included in the stable closed-loop system to achieve zero tracking error to a step reference. On the other hand, there are many applications, such as voltage-source pulsewidth modulated (PWM) inverters, in which the reference signals to be tracked and/or the disturbances to be rejected are periodic signals, with harmonic components of a common fundamental frequency. For these applications, a periodic signal generator, which is represented in discrete time by poles harmonically placed around the unit circle, must be included in the closed-loop system to eliminate the steady-state error.

In the discrete time domain, periodic signals with a known period T can be generated by a time delay block with a positive feedback loop as illustrated in Fig. 1 [2], [3]. This system has n poles equally distributed on the unit circle and, therefore, it is possible to track a periodic reference or reject periodic disturbances with the same fundamental frequency. A controller including this model is said to be a *repetitive controller* [2].

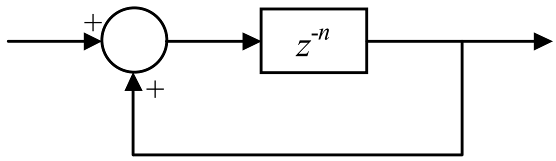

Fig. 1 – Periodic signals generator ($n = T/T_s$, T_s = sampling frequency).

As a result, repetitive controllers have been widely employed as an alternative to minimize periodic errors that can occur in a dynamic system. Several modified repetitive control schemes have been developed and applied in various industrial applications [3]-[5]. Repetitive control theory has also been applied to voltage-source PWM inverters, which are employed in UPS (Uninterruptible Power Supply) systems, to minimize the steady-state error and periodic distortions caused by nonlinear cyclic loads, such as rectifier-type loads [6]-[10]. The output voltages synthesized by repetitive controlled PWM inverters present very low THD (Total Harmonic Distortion) even under severe nonlinear loads (rectifier RC load with a current crest factor of 3). However, as repetitive controller is a learning controller that uses the information of the output error in the previous cycles to compute the repetitive action, the conventional repetitive controller does not present a good dynamic performance under non-periodic disturbances, such as sudden linear load changes or removal of a nonlinear cyclic load.

Therefore, this paper proposes an algorithm for improving the transient response of repetitive controlled voltage-source PWM inverters under these non-periodic disturbances. The proposed algorithm is based on the analysis of the output error behavior to identify the occurrence of non-periodic disturbances. Once identified the occurrence of a non-periodic disturbance, it is possible to reset the repetitive control action so that the repetitive controller can storage the correct information about this new load.

This paper is organized as follows: Section II describes the plant to be controlled and Section III presents the structure of the digital control system. Section IV analyses the problem of the repetitive controller dynamic performance under non-periodic disturbances and proposes an improved repetitive controller algorithm to overcome this problem. Section V presents some simulation results that show the transient performance of the proposed algorithm and compares it with the conventional repetitive controller. Finally, Section VI shows some experimental results (1 kVA @ 110 V_{RMS}) to verify the feasibility of the proposed control algorithm.

II. PLANT MODEL

A single-phase voltage-source PWM inverter is illustrated in Fig. 2, where the full-bridge inverter, LC filter, and load are considered as the plant to be controlled. Moreover, a single-phase diode bridge rectifier with capacitive filter can be used to evaluate the performance of the system with nonlinear loads.

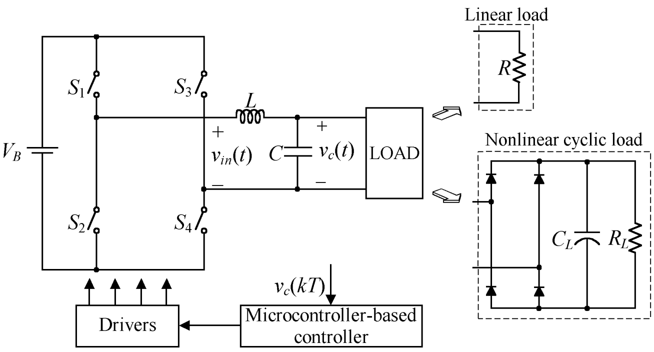

Fig. 2 – Digitally controlled PWM inverter.

Due to diversity of loads, it is not possible to formulate a general model to cover every kind of load. In this case, a nominal load is defined to derive a linear model, and then load variations and model uncertainties are considered as load disturbances. In addition, given the assumption that the switching frequency is much higher than the modulation frequency of the PWM inverter, the system presented in Fig. 2 can be modeled as a linear second-order system. Thus, the transfer function of the system shown in Fig. 2 is:

$$\frac{Y(s)}{U(s)} = G_p(s) = \frac{\omega_p^2}{s^2 + 2\zeta_p \omega_p s + \omega_p^2} \tag{1}$$

where $\omega_p = 1/\sqrt{LC}$, $\zeta_p = 1/(2RC\omega_p)$, $Y(s)$ is defined as the Laplace transform of the system output $y(t) = v_c(t)$ and $U(s)$ is defined as the Laplace transform of the system input $u(t) = v_{in}(t)$.

The power switches are turned on and off once during each sampling interval T_s, such that $v_{in}(t)$ is a voltage pulse of magnitude V_B or $-V_B$, and width ΔT. However, in the following analysis, it is assumed that $u(t)$ is the average value of the voltage pulse in a sampling period. Thus, a discrete transfer function can be obtained from (1) using a zero-order hold with an appropriate sampling time T_s [11]:

$$G_p(z) = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2}. \tag{2}$$

III. STRUCTURE OF THE CLOSED-LOOP SYSTEM

Fig. 3 presents a simplified structure of the digital control system employed to control the single-phase PWM inverter shown in Fig. 2. The digital control law, which is composed of two terms, is given by:

$$u(k) = u_C(k) + u_{RP}(k). \tag{3}$$

A conventional feedback controller (PD controller, deadbeat controller, state feedback control, etc.) is utilized to improve

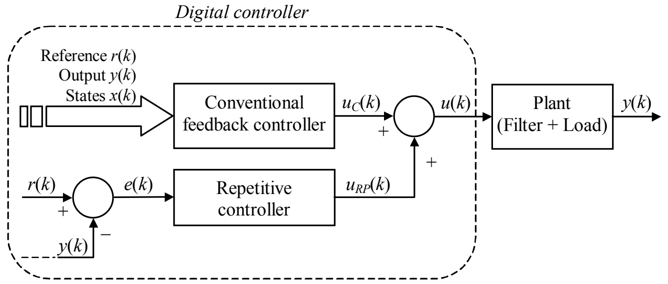

Fig. 3 – Digital control system with repetitive controller.

the dynamic response and/or to increase the stability margin of the closed-loop system. However, these controllers can present poor results for periodic disturbances, such as nonlinear cyclic loads or low frequency (120 Hz) dc bus voltage ripple. Thus, a repetitive controller is included to the control system to minimize distortions caused by periodic disturbances.

As mentioned before, repetitive control theory is based on internal model principle [1], which affirms that the plant output tracks the reference signal without steady-state error if the model that generates this reference is included in stable closed-loop system. In UPS applications, a periodic input or disturbance may consist of fundamental and high-order harmonics. In this way, the repetitive controller should be able to include the model of these signals.

In a similar way to that presented in [8], the following repetitive controller transfer function, which can generate periodic signals consisting of harmonic components of a common fundamental frequency, can be used:

$$\frac{u_{RP}(z)}{e(z)} = \frac{c_r z^{N-n}}{1 - Q_r(z) z^{-n}} \tag{4}$$

where $e(z)$ is the z-transform of the error $e(k) = r(k) - y(k)$, c_r is the gain of the repetitive controller, N is the time advance step size, n is the number of samples in a reference voltage period and $Q_r(z)$ is the transfer function of a low pass filter or a constant equal or little smaller than unit. The term $Q_r(z)$ has been included to the repetitive controller to improve the robustness of the closed-loop system.

Therefore, assuming that $Q_r(z)$ is a constant, the repetitive control law can be written as:

$$u_{RP}(k) = c_r e(k + N - n) + Q_r u_{RP}(k - n). \tag{5}$$

If $Q_r(z)$ is equal to unit, steady-state errors caused by periodic signals and disturbances could be completely eliminated. However, the robustness of the closed-loop system is improved if a constant little smaller than unit is used. By using a smaller value for $Q_r(z)$, it is possible to increase the stability margin. Nevertheless, low frequency harmonic components will not be adequately rejected, increasing the output voltage THD even under periodic disturbances. On the other hand, the choice of the repetitive controller gain c_r should ensure fast convergence of the output error and maintain the closed-loop system stable. Higher repetitive controller gain results in fast convergence, but the feedback system may become unstable for large values of c_r.

IV. DESCRIPTION OF THE PROBLEM AND PROPOSED SOLUTION

From (5), it is possible to verify that repetitive controller is a learning controller that uses the information of the output error in the previous cycles to compute the control action. Based on these information, the repetitive controller gradually reduces distortions caused by periodic disturbances. Therefore, with the appropriate design of the repetitive controller, the closed-loop system usually presents a good steady-state performance when reference signal and disturbances are periodic. However, the conventional repetitive controller presents a poor transient response for non-periodic disturbances, such as linear load changes or after remove a nonlinear cyclic load.

For instance, by applying a sudden load change, the output voltage waveform will present a distortion caused by the voltage drop on the filter inductance. In the next cycle of the output voltage waveform, the repetitive action will attempt to compensate this error, but, due to the non-periodicity of this disturbance, the output voltage waveform will be distorted by the repetitive control action. In a similar way, in the first cycle after remove a nonlinear cyclic load, the repetitive control action will inject an unnecessary energy in the plant and, therefore, it will degrade the output voltage waveform in the next cycles.

Consequently, it is necessary to modify the repetitive control algorithm so that the type of disturbance can be identified. Once identified the occurrence of a non-periodic disturbance (linear load change or removal of a nonlinear cyclic load), the repetitive control action should be reset so that the output voltage waveform in the next cycles will not be distorted.

When a periodic disturbance is applied to the PWM inverter, the output error gradually converges to zero. However, by applying a non-periodic disturbance, the absolute value of the output error in the point when occurred this disturbance will be greater than the absolute value of the error in the same point of the previous cycle. Therefore, an improved repetitive control algorithm to improve the transient response under non-periodic disturbances can use this concept and it is given below:

$$\begin{aligned}
&\textit{if } \; |e(k)| - |e(k-n)| > e_{\lim}, \\
&\textit{then } \; \begin{cases} u_{RP}(k) = 0 \\ u_{RP}(k+1) = 0 \\ \quad \vdots \\ u_{RP}(k+n) = 0 \end{cases}; \\
&\textit{else } \; u_{RP}(k) = c_r e(k+N-n) + Q_r u_{RP}(k-n)
\end{aligned} \tag{6}$$

where $e_{\lim}$ is a specified limit value of the difference between the absolute value of the output error in the point that occurred the disturbance and the absolute value of the error in the same point of the previous cycle. This limit value is a greater than zero constant, which ensures that repetitive control action will not be reset by quantization errors and measurement noises.

V. SIMULATION RESULTS

This section presents simulation results obtained with MATLAB®, which show the good steady-state performance and the poor dynamic performance of the conventional repetitive controller under non-periodic disturbances. Moreover, other simulation results are given to verify that improved repetitive controller presents a good transient response even under non-periodic disturbances. The conventional feedback control employed to obtain these results is the predictive PD-feedforward controller [10], which measures only the output voltage and whose control law is given by:

$$u_C(k) = K_1 e(k-1) + K_2 e(k-2) + r(k)\,. \tag{7}$$

It is important to point out that any feedback controller can be utilized to improve the dynamic performance and/or to increase the stability margin of the closed-loop system, because the proposed repetitive control algorithm is independent from the conventional feedback controller.

The parameters of the single-phase voltage-source PWM inverter are shown in Table I and the parameters of the controller are given in Table II.

TABLE I - PARAMETERS OF PWM INVERTER.

Filter inductance	$L = 1$ mH
Filter capacitance	$C = 25$ µF
DC input voltage	$V_B = 200$ V
Reference voltage	$r = 110$ V_{RMS}, $f = 60$ Hz
Nominal resistive load	$R = 12$ Ω (1 kVA)
Crest factor of the nonlinear load	$CF \cong 3$
Sampling frequency	$f_S = 10800$ Hz
Sampling time	$T_S = 92.6$ µs

TABLE II - PARAMETERS OF THE CONTROLLER.

Predictive PD-feedforward controller	$K_1 = 0.1033$ $K_2 = -0.2523$
Repetitive controller	$c_r = 0.25$ $Q_r = 0.98$ $N = 3$

Fig. 4 shows the output voltage (THD = 10.5 %), load current and reference voltage (dotted line) waveforms for a rectifier load, without use the repetitive controller. Fig. 5 also presents the output voltage (THD = 1.19 %) and load current

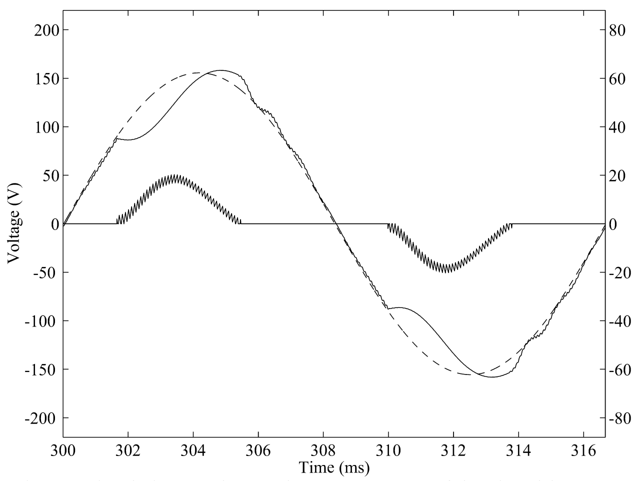

Fig. 4 – Simulation result. Steady-state response of the closed-loop system without repetitive controller under a rectifier load.

waveforms for the same nonlinear cyclic load by including the repetitive control action. From Fig. 4 and Fig. 5 it is possible to observe that repetitive control action decreases significantly the output voltage distortion for this nonlinear cyclic load (periodic disturbance).

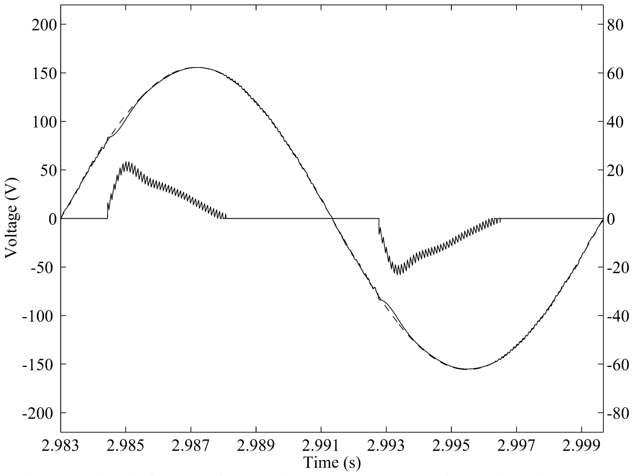

Fig. 5 – Simulation result. Steady-state response of the closed-loop system with repetitive controller under a rectifier load.

Fig. 6 shows the response of the digital control system with conventional repetitive control action under a sudden load change from no-load to full-load. Fig. 6(a) presents the output voltage and load current waveforms and Fig. 6(b) gives the repetitive control action $u_{RP}(k)$ waveform. From Fig. 6(b) it is possible to verify that the repetitive action uses the value of the output error in the point that occurred the linear load change and, therefore, it attempts to reduce this error in the next cycle. Nevertheless, as this disturbance does not occur again, the output voltage waveform is distorted and dynamic performance is damaged, as illustrated in Fig. 6(a).

On the other hand, Fig. 7 gives the response of the control system with the improved repetitive controller, by applying the same linear load change of Fig. 6. Once identified the occurrence of the non-periodic disturbance, the repetitive control action is reset as shown in Fig. 7(b). Thus, the repetitive control action will not attempt to compensate an inexistent periodic disturbance and the dynamic performance is improved, as illustrated in Fig. 7(a).

Moreover, Fig. 8 shows the transient response of the digital control system with conventional repetitive controller after remove a rectifier load (nonlinear cyclic load). The

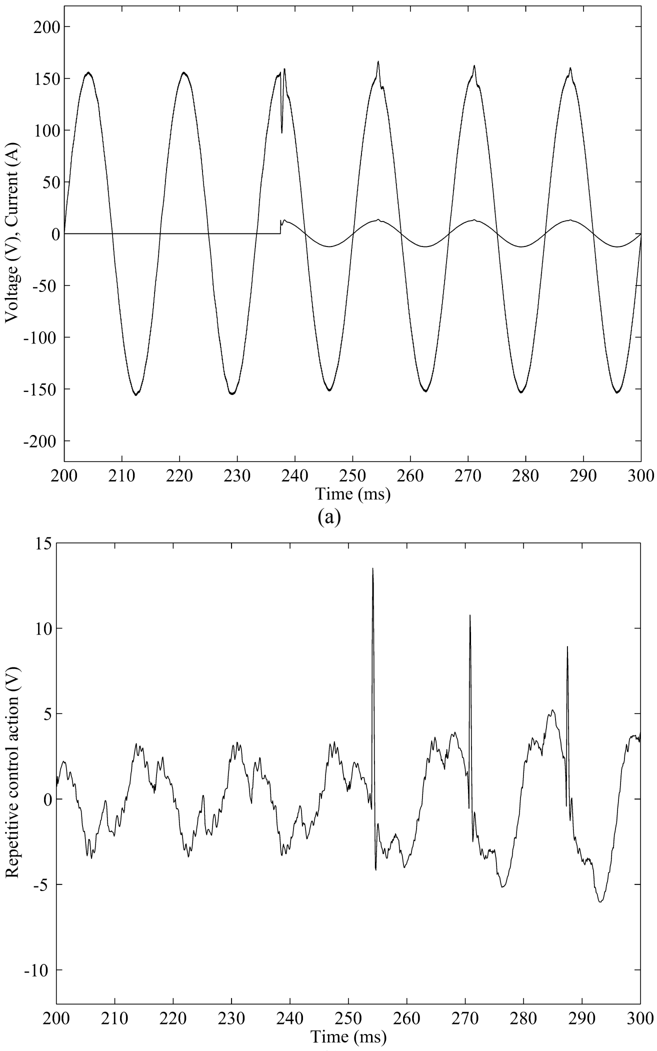

Fig. 6 – Simulation result. Transient response of the conventional repetitive controller under a sudden load change: no-load to full-load. (a) Output voltage and load current. (b) Repetitive control action.

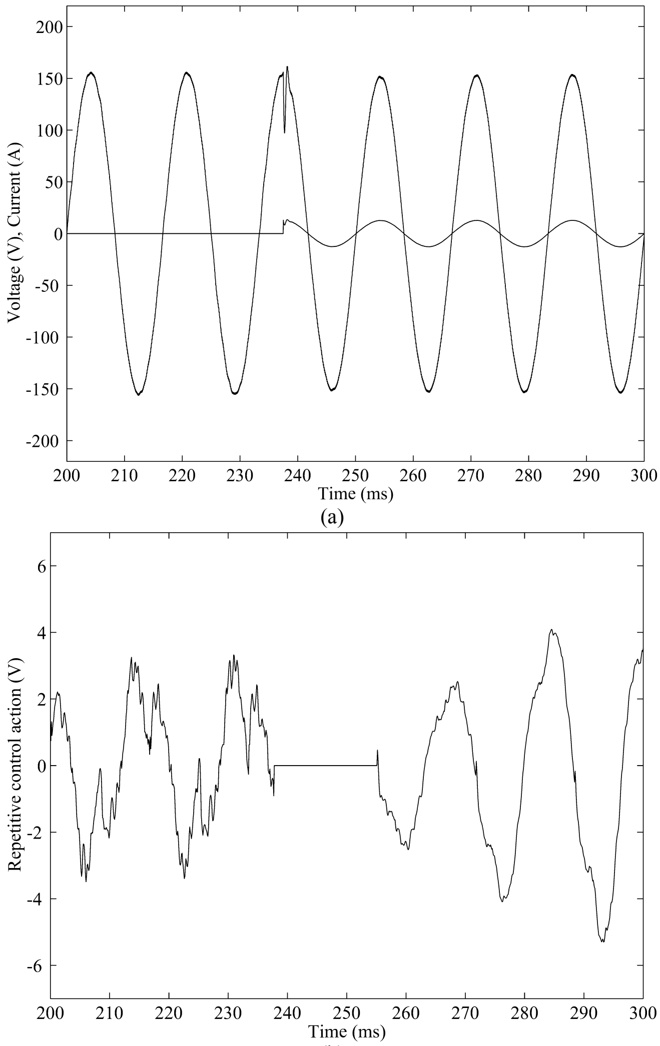

Fig. 7 – Simulation result. Transient response of the improved repetitive controller under a sudden load change: no-load to full-load. (a) Output voltage and load current. (b) Repetitive control action.

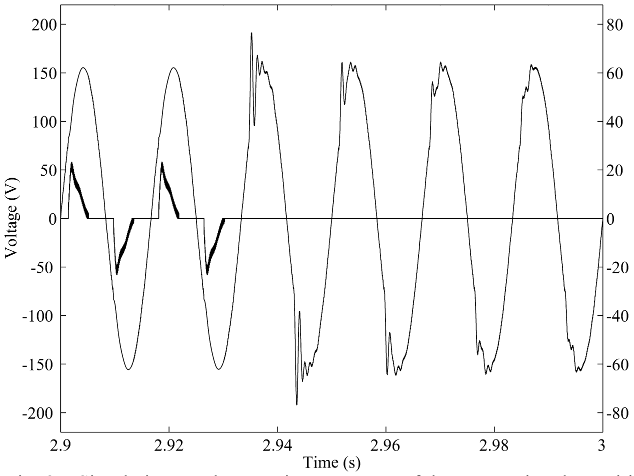

Fig. 8 – Simulation result. Transient response of the conventional repetitive controller after remove a rectifier load.

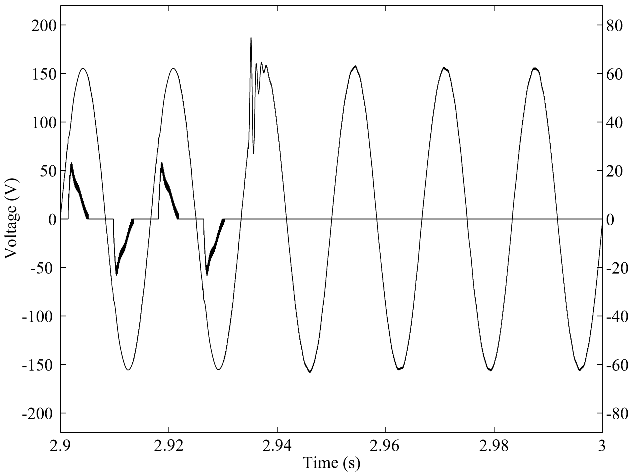

Fig. 9 – Simulation result. Transient response of the improved repetitive controller after remove a rectifier load.

repetitive control action attempt to compensate the removed periodic disturbance, damaging the performance of the PWM inverter system. Fig. 9 presents the dynamic performance of the closed-loop system with the improved repetitive controller for the same case illustrated in Fig. 8. In this case, after identify this new load condition, the repetitive control action is reset and, thus, the transient response of the closed-loop system is significantly improved.

VI. Experimental Results

A laboratory prototype of a single-phase PWM inverter using IGBTs has been built to illustrate the transient performance of the closed-loop system with a repetitive controller. The component values of the inverter system and the parameters of the controller are the same used in simulation (Tables I and II). The simplified block diagram of the experimental setup is shown in Fig. 2. The controller has been implemented using an 8-bit data word microcontroller (PIC17C756 of Microchip Technology Inc.), which has an embedded 10 bits A/D converter and a PWM signal generator that reduce the PWM inverter control circuitry.

Fig. 10 shows the output voltage (THD = 1.25%) and load current waveforms for a rectifier load with a current crest factor around 3. From this figure, it is possible to verify the excellent steady-state performance of the closed-loop system with repetitive controller, even under this severe nonlinear load.

On the other hand, Fig. 11 presents the transient response of the conventional repetitive controller and Fig. 12 shows the dynamic performance of the improved repetitive controller under a similar sudden load change from no-load

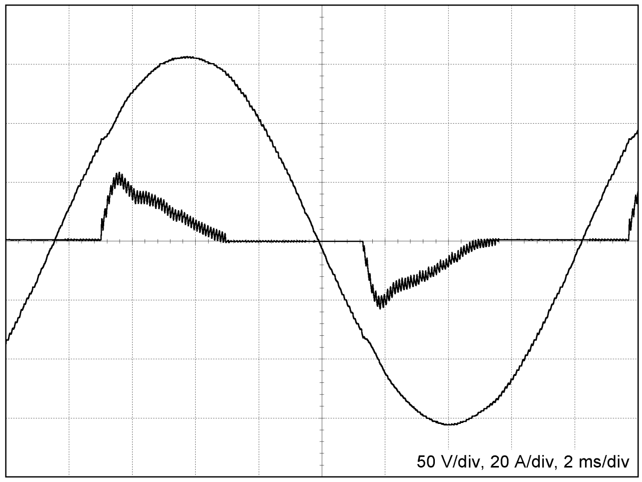

Fig. 10 – Experimental result. Steady-state response with a rectifier load.

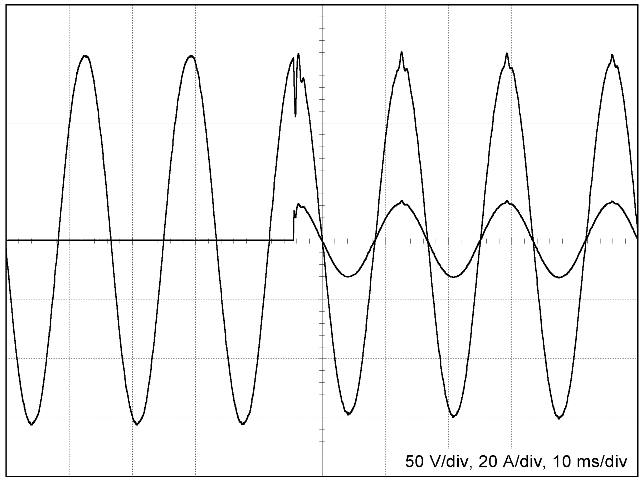

Fig. 11 – Experimental result. Transient response of the conventional repetitive controller under a sudden load change: no-load to full-load.

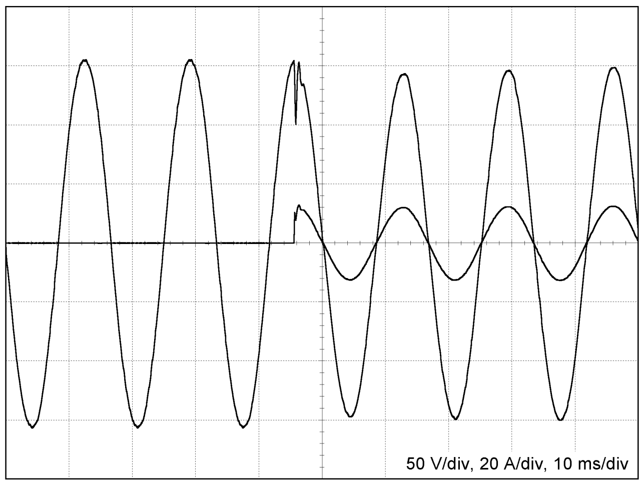

Fig. 12 – Experimental result. Transient response of the improved repetitive controller under a sudden load change: no-load to full-load.

to full load. As mentioned before, once identified the occurrence of the non-periodic disturbance, the repetitive control action is reset for one line period. Thus, the repetitive control action will not attempt to compensate an inexistent periodic disturbance and the dynamic performance is improved, as depicted in Fig. 12.

Finally, Fig. 13 shows the transient response of the conventional repetitive controller and Fig. 14 presents the dynamic performance of the improved repetitive controller after remove a rectifier load with a current crest factor around 3. Again, the occurrence of the non-periodic disturbance is correctly identified and, therefore, the repetitive control action is reset. Consequently, the dynamic performance is significantly improved, as shown in Fig. 14.

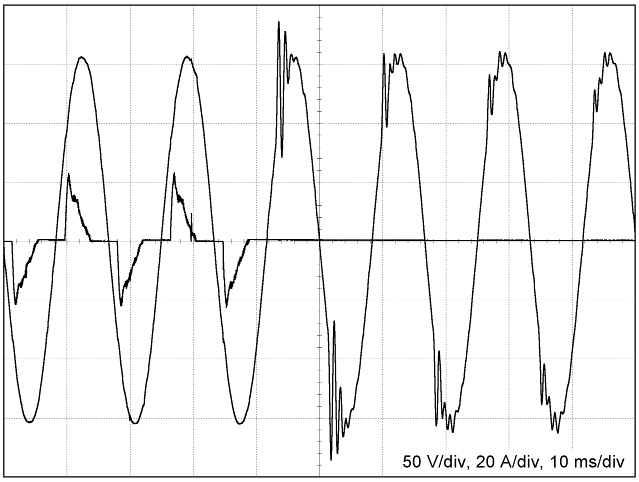

Fig. 13 – Experimental result. Transient response of the conventional repetitive controller after remove a rectifier load.

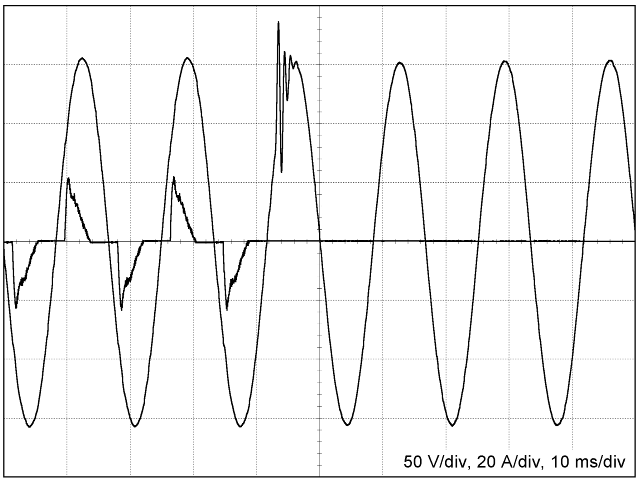

Fig. 14 – Experimental result. Transient response of the improved repetitive controller after remove a rectifier load.

VII. CONCLUSIONS

Although the closed-loop system with repetitive controller presents a good steady-state performance when reference signal and disturbances are periodic, the conventional repetitive controller has a poor transient response under non-periodic disturbances, such as a sudden linear load change or after remove a nonlinear cyclic load. Therefore, this paper proposed an algorithm for improving the dynamic performance of repetitive controlled PWM inverters under non-periodic disturbances. The proposed algorithm is based on the analysis of the output error behavior to identify the occurrence of non-periodic disturbances. Once identified the occurrence of a non-periodic disturbance, the repetitive control action is reset so that the repetitive controller storages the correct information concerning this new load. Experimental results show that the proposed repetitive control algorithm improves significantly the dynamic performance of voltage-source PWM inverters.

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