ROBUST DISCRETE INPUT-TO-STATE CONTROL OF THE INDUCTION MOTOR

Jesus Franklin Andrade Romero and Elder Moreira Hemerly
Division of Electronic Engineering
Department of Systems and Control
Technological institute of Aeronautics
ITA - IEE - IEES, São José dos Campos - SP - Brazil
jesus@ele.ita.cta.br and hemerly@ele.ita.cta.br

Abstract— **In this work a global adaptive feedback control system for induction motor servo drives is proposed. A certainty equivalence approach is used based on nonlinear discrete input-to-state (ISS) and robust stability concepts [1]. The control algorithm design is based on a continuous feedback stabilization technique proposed in Marino et al. [2] and the adaptive feedback control is based on the discrete extensions of the input-to-state and robust stability definitions established in Sontag et al. [3]. The controller is based on the indirect field oriented characteristic equations. A convergent flux observer, Kalman extended filter based [4], is used to implement the control feedback loop and to identify the rotor resistance value. Different operation conditions and practical implementation characteristics of the system (PWM algorithms and antialiasing filters) are considered in several simulations.**

Keywords— **Nonlinear Systems, Stability Analysis, Deterministic Discrete-Time Systems, Input-to-State Stability, Robust stability, Induction Motor Drives**

I. Introduction

The output feedback problem for the nonlinear system shaped by the induction motor model and the control system (for flux and speed regulation) presents several approaches and a great variety of solutions has been proposed.

Assuming no parametric variations and that all the state variables (including rotor fluxes) are available, the regulation problem of flux and speed can be solved by the field oriented control [5], or by the input-output linearizing control. In the second approach, the nonlinear state-space change of coordinates and the nonlinear state feedback transformation are used for a more ambitious objective, the exact feedback variables decoupling, i.e., the speed and flux modulus can be independently controlled with linear dynamics. The field oriented control achieves the same property only asymptotically, provided that the reference for the flux modulus is kept constant.

Considering the magnitude of the parametric variations in the rotor resistance and load torque variables, in different operation states, the control technique must solve this problem through an adaptive or robust approach [6], [7].

In this work a discrete indirect field oriented based controller is presented. The control design provides a stability proof when no parametric variations in the motor are assumed and all state variables (rotor fluxes) are available. Also, a global adaptive feedback control system is proposed using a certainty equivalence approach based on nonlinear discrete input-to-state (ISS) and robust stability concepts [3], [1]. A nonlinear asymptotic observer proposed in [4] is used for rotor flux estimation and rotor resistance identification.

The controller design, one of the contributions of the work, is based on a continuous feedback stabilization technique proposed in [2]. It must be pointed out that it is assumed a current-fed machine and the observer is based on the reduced-order model of the IM, thus, there is no need for considering the stator circuit dynamics and the computational cost of the controller implementation is decreased.

The IM model and the regulation errors are defined in section II. The discrete feedback controller is described in section III. The design of the nonadaptive version of the stabilization controller is presented in section IV. The stability considerations, assuming parametric variations, is presented in section V. In section VI, some simulations are presented and the observability analysis of the reduced IM model when assuming rotor resistance identification are made. The concluding remarks are presented in section VII.

II. Problem Formulation

The reduced induction motor model, equation (III.15), considering a general (dq) time varying reference frame rotating at speed w_ϕ, becomes

$$\Phi_{Rdq(k+1)}=\left[\left(1-\frac{T_aR_R}{L_R}\right)I-T_a(w_{\phi(k)}-w_{m(k)})J_c\right]\Phi_{Rdq(k)}+\frac{T_aLmR_R}{L_R}\mathrm{I}_{Sdq(k)} \quad (1)$$

where $\Phi_{Rdq(k)}=\left[\phi_{Rd(k)},\phi_{Rq(k)}\right]^T$ and $i_{Sdq(k)}=\left[i_{sd(k)},i_{sq(k)}\right]^T$ represent the rotor flux and stator currents vectors, respectively. T_a represent the sampling time, I represent the dimension 2 identity matrix, J_c the matrix defined as $J_c=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ and $0_{2\times2}$ the null matrix.

The discrete IM model is complemented by the mechanical dynamic, more precisely,

$$w_{m(k+1)}=\left(1-\frac{DT_a}{J}\right)w_{m(k)}+\frac{T_anLm}{JL_R}i_{sq(k)}\phi_{Rd(k)}\frac{T_anLm}{JL_R}i_{sd(k)}\phi_{Rq(k)}-\frac{T_a}{J}T_{L(k)} \quad (2)$$

where the electric torque generated is $T_{(k)}=\frac{nL_m}{L_R}\left(i_{sq(k)}\phi_{Rd(k)}-i_{sd(k)}\phi_{Rq(k)}\right)$, J represent the moment of inertia, n the number of pole pairs in the motor, D the coefficient of viscous friction and T_L the load torque.

Let us denote by w_{mref} and Φ_{ref} the smooth bounded references signals for the output variables to be controlled, i.e. speed and rotor flux modulus. Considering the field oriented control strategy [5], the tracking errors of the speed and the (d,q) flux components are defined as

$$\begin{aligned}\widetilde{w}_{m(k)} &= w_{m(k)}-w_{mref(k)} && (3)\\ \widetilde{\phi}_{Rd(k)} &= \phi_{Rd(k)}-\Phi_{ref(k)} && (4)\\ \widetilde{\phi}_{Rq(k)} &= \phi_{Rq(k)} && (5)\end{aligned}$$

Notice that aiming a discrete dynamic feedback compensator and considering the rotor flux field oriented principle, the technique must guarantee that $\dot{\rho}_{(k)}=w_{\phi(k)}$ and

$$\begin{bmatrix} i_{s\alpha(k)}\\ i_{s\beta(k)}\end{bmatrix}=\begin{bmatrix}\cos\left(\rho_{(k)}\right) & \text{-}\sin\left(\rho_{(k)}\right)\\ \sin\left(\rho_{(k)}\right) & \cos\left(\rho_{(k)}\right)\end{bmatrix}\begin{bmatrix} i_{sd(k)}\\ i_{sq(k)}\end{bmatrix}$$

by means of the measured variables ($w_{m(k)}$, $i_{sdq(k)}$, $v_{sdq(k)}$). Then, for any unknown $T_{L(k)}$ and $R_{R(k)}$, and for any initial condition ($w_{m(0)}$, $\phi_{R\alpha\beta(0)}$, $i_{s\alpha\beta(0)}$) we obtain

$$\begin{aligned}\lim_{k\to\infty}\left(w_{m(k)}-w_{mref(k)}\right) &= 0 \quad (6)\\ \lim_{k\to\infty}\left(\phi_{rd(k)}-\Phi_{ref(k)}\right) &= 0\\ \lim_{k\to\infty}\phi_{rq(k)} &= 0\end{aligned}$$

which imply

$$\lim_{k\to\infty}\left(\sqrt{\phi_{ra(k)}^2+\phi_{rb(k)}^2}-\Phi_{ref(k)}\right)=0 \quad (7)$$

It must be pointed out that conditions (6) and (7) clearly implies rotor flux field orientation.

III. Discrete Feedback Controller

As in [2] lets proceed, initially, with the design of the global discrete dynamic speed feedback controller assuming no parametric variations, i.e., lets consider the discrete reduced order model of the IM, equation (1), assuming both T_L and R_R as known. Basing our approach in the indirect field oriented control [8], conditions (6) and (7) are achieved when

$$\begin{aligned}w_{\phi(k)} &= w_{m(k)}+\frac{R_RL_m}{L_R\Phi_{ref(k)}}i_{sq(k)}-\frac{u_{sq(k)}}{\Phi_{ref(k)}} && (8)\\ i_{sd(k)} &= \frac{L_R}{R_RT_aL_m}(\phi_{ref(k)}-\phi_{ref(k-1)})\\ &\quad+\frac{\phi_{ref(k-1)}}{L_m}+\frac{L_R}{R_RL_m}u_{sd(k)} && (9)\\ i_{sq(k+1)} &= \frac{JL_R}{nL_m\Phi_{ref(k)}}\Big(\frac{D}{J}w_{m(k)}-k_2\widetilde{w}_{m(k)}+\frac{T_{l(k)}}{J}\\ &\quad+\frac{w_{mref(k+1)}-w_{mref(k)}}{T_a}+\frac{u_{w(k)}}{T_a}\Big) && (10)\end{aligned}$$

where $k_2>0$ is a design control parameter that allows fast speed regulation and $(u_{sd(k)},u_{sq(k)},u_{w(k)})$ are suitable functions to be used for the stability proof .

Remark 1. It must be pointed out that equations (8), (9) and (10) are determined based on the discretizated representative equations of the continuous indirect field oriented control. The main objective of these expressions is to guarantee cancellation of terms that forces the rotor flux magnitude to be close to the reference value and to allow an appropriate speed regulation.

Remark 2. For continuous time systems, continuous stabilization implies ISS stabilization by means of a state-feedback change $u=K(x)+v$. For discrete-time systems, a more complex feedback transformation of the form $u=K_1(x)+K_2(x)v$ is required in general. The constructions of the feedback terms K_1 and K_2 turns out to be nontrivial [3].

Now, subtracting the IM reduced model, equations (1)-(2), by proper controller expressions, (8)-(10), we obtain the discrete dynamics of the regulation error, i.e.,

$$\widetilde{w}_{m(k+1)} = (1-k_2T_a)\widetilde{w}_{m(k)} + \frac{T_a n L_m}{J L_R}$$
$$(i_{sq(k)}\widetilde{\phi}_{rd(k)} - i_{sd(k)}\widetilde{\phi}_{rq(k)}) + u_{w(k)} \quad (11)$$
$$\widetilde{\phi}_{Rd(k+1)} = \widetilde{\phi}_{Rd(k)} - \frac{TaR_R}{L_R}\widetilde{\phi}_{Rd(k)}$$
$$+T_a(w_{\phi(k)} - w_{m(k)})\widetilde{\phi}_{Rq(k)} + T_a u_{sd(k)} \quad (12)$$

$$\widetilde{\phi}_{Rq(k+1)} = \widetilde{\phi}_{Rq(k)} - \frac{TaR_R}{L_R}\widetilde{\phi}_{Rq(k)}$$
$$-T_a(w_{\phi(k)} - w_{m(k)})\widetilde{\phi}_{Rd(k)} + T_a u_{sq(k)} \quad (13)$$

IV. Stability Analysis

Before proceeding with the design of the discrete controller assuming parametric variations, some central results obtained in [9] and [3] are summarized. Consider the general nonlinear discrete time system,

$$x_{(k+1)} = f\left(x_{(k)}, u_{(k)}\right) \quad (14)$$

where states $x_{(k)}$ are in $\Re^n$, and control values $u_{(k)}$ in $\Re^m$, for some n and m, for each time instant k$\in \jmath_+$, the set of all no negative integers. We assume that $f : \Re^n \times \Re^m \to \Re^n$ is continuous and satisfies $f(0,0) = 0$.

Definition 1. A function $\gamma : \Re_{\geq 0} \to \Re_{\geq 0}$ is a K-function if it is continuous, strictly increasing and $\gamma(0) = 0$; it is a $K_\infty - function$ if it is a $K - function$ and also $\gamma(s) \to \infty$ as $s \to \infty$[9].

Definition 2. A continuous function V on $\Re^n$ is called an ISS-Lyapunov function for the system (14) if

$$\alpha_1(|\xi|) \leq V(\xi) \leq \alpha_2(|\xi|)$$

holds for some α_1, $\alpha_2 \in K_\infty$, and

$$V(f(\xi,\mu)) - V(\xi) \leq -\alpha_3(|\xi|) + \sigma(|\mu|) \quad (15)$$

for some $\alpha_3 \in K_\infty$, and $\sigma \in K$[3]. A smooth ISS-Lyapunov function is one which is smooth.

Theorem 1. If system (14) admits an ISS-Lyapunov function, then it is input-to-state stable (ISS).

Proof. See [3] ■

Now, suppose there is a continuous function $\gamma(\cdot)$ that belongs to class *K-function*, such that

$$V_{k+1} - V_k \leq -\gamma\left(\left\|\widetilde{w}_{m(k)}\right\|, \left\|\widetilde{\phi}_{Rd(k)}\right\|, \left\|\widetilde{\phi}_{Rq(k)}\right\|\right) < 0 \quad (16)$$

then the sequence $\{V_k\}$ is strictly decreasing. In order to verify if $\{V_k\}$ is indeed strictly decreasing, let $\gamma\left(\left\|\widetilde{w}_{m(k)}\right\|, \left\|\widetilde{\phi}_{Rd(k)}\right\|, \left\|\widetilde{\phi}_{Rq(k)}\right\|\right) = \delta V_k$. Then the input-to-state stability proof for the discrete system, (8)-(10), can be obtained and is summarized in one lemma and one theorem.

Lemma 1. Let the design input functions ($u_{sd(k)}$, $u_{sq(k)}$, $u_{w(k)}$) be given by the following expressions,

$$u_{w(k)} = -\gamma_1\left[\frac{T_a n L_m}{J L_R}\left(i_{sq(k)}\widetilde{\phi}_{Rd} - i_{sd(k)}\widetilde{\phi}_{rq(k)}\right) + 2\left(k_2T_a - 1\right)\widetilde{w}_{mk}\right] \quad (17)$$
$$u_{sd(k)} = -\gamma_1 \frac{T_d\widetilde{\phi}_{Rd(k)} + T_aT_R w_{sl(k)}\widetilde{\phi}_{Rq(k)}}{T_aT_R} \quad (18)$$
$$u_{sq(k)} = \gamma_1 \frac{-T_aT_d\widetilde{\phi}_{Rq(k)} + T_aT_R w_{sl(k)}\widetilde{\phi}_{Rd(k)}}{T_aT_R} \quad (19)$$

where $T_d = T_R - T_a$, $w_{sl(k)} = w_{\phi(k)} - w_{m(k)}$ and γ_1 is a positive design parameter. Then, by defining the following Lyapunov candidate function,

$$V_{(k+1)} = \left(\widetilde{w}_{m(k+1)} + \left(\frac{1-\gamma_1}{\gamma_1}\right)u_{w(k)}\right)^2 + \left(\widetilde{\phi}_{Rd(k+1)} + \left(\frac{1-\gamma_1}{\gamma_1}\right)T_a u_{sd(k)}\right)^2 + \left(\widetilde{\phi}_{Rq(k+1)} + \left(\frac{1-\gamma_1}{\gamma_1}\right)T_a u_{sq(k)}\right)^2 \quad (20)$$

the regulation errors $(\widetilde{w}_{(k)}, \widetilde{\phi}_{Rd(k)}, \widetilde{\phi}_{Rq(k)})$ converges to zero asymptotically when δ, k_2 and γ_1 satisfy the following inequalities,

$$0 < \delta < 1 \quad (21)$$
$$\frac{1-\sqrt{1-\delta}}{Ta} \leq k_2 \leq \frac{1+\sqrt{1-\delta}}{T_a} \quad (22)$$
$$(1-\gamma_1)(1-\delta)[(1-\gamma_1)u^2_{w(k-1)} + 2\widetilde{w}_k u_{w(k-1)}\gamma_1] \geq 0 \quad (23)$$

Remark 3. Since the inputs variables $u_{w(k)}, u_{sd(k)}$ and $u_{sq(k)}$ are explicit functions of the states, and $\left(u_{w(k)}, u_{sd(k)}, u_{sq(k)}\right) = 0$ when $\left(\widetilde{w}_{m(k)}, \widetilde{\phi}_{Rd(k)}, \widetilde{\phi}_{Rq(k)}\right) = 0$, then, $V_{(0)} = 0$. Moreover, once $V_{(k)}$ presents a quadratic form, then, there exist class κ_∞ functions α_1 and α_2, such that

$$\alpha_1(\|x_k\|) \leq V(x_k) \leq \alpha_2(\|x_k\|)$$

for all $x_k \in \Re^n$ [9].

Remark 4. The main purpose of $\left(\frac{1-\gamma_1}{\gamma_1}\right)$ terms in (20) is to force the positive design parameter γ_1 in the input expressions (17)-(19).

Proof. Substituting the regulation error dynamics, (11)-(13), and the design functions expressions, (17)-(19), into the Lyapunov candidate function

(20) it is possible to verify that

$$V_{(k+1)}-(1-\delta)V_{(k)}=(k_2^2T_a^2-2k_2T_a+\delta)\widetilde{w}_{m(k)}^2$$
$$-\frac{(1-\gamma_1)(1-\delta)}{\gamma_1^2}$$
$$\left[(1-\gamma_1)u_{w(k-1)}^2+2\widetilde{w}_{mk}u_{w(k-1)}\gamma_1\right]$$
$$-(1-\delta)\left[\widetilde{\phi}_{Rd(k)}+\left(\frac{1-\gamma_1}{\gamma_1}\right)T_au_{sd(k-1)}\right]^2$$
$$-(1-\delta)\left[\widetilde{\phi}_{Rq(k)}+\left(\frac{1-\gamma_1}{\gamma_1}\right)T_au_{sq(k-1)}\right]^2 \quad (24)$$

As we can note, the inequality (16) is satisfied when conditions (21)-(23) are guaranteed. Therefore, $(\widetilde{w}_{(k)},\widetilde{\phi}_{Rd(k)},\widetilde{\phi}_{Rq(k)})=0$ *is a globally exponentially stable equilibrium point for the closed-loop system (17)-(19) and (11)-(13).* ■

The final expressions for the discrete dynamic compensator are obtained substituting equations (17)-(19) into the system input variables, (8), (10), more precisely

$$w_{\phi(k)}=w_{m(k)}+\frac{L_m}{T_R(\Phi_{ref(k)}+\gamma_1\widetilde{\phi}_{Rd(k)})}i_{sq(k)}$$
$$+\frac{T_d}{T_aT_R}\frac{\gamma_1\widetilde{\phi}_{Rq(k)}}{\Phi_{ref(k)}+\gamma_1\widetilde{\phi}_{Rd(k)}} \quad (25)$$
$$i_{sd(k)}=\frac{L_R}{R_RT_aL_m}(\phi_{ref(k)}-\phi_{ref(k-1)})$$
$$+\frac{\phi_{ref(k-1)}}{L_m}-\frac{T_d}{T_aL_m}\gamma_1\widetilde{\phi}_{Rd(k)}$$
$$-\frac{\gamma_1\widetilde{\phi}_{Rq(k)}}{\Phi_{ref(k)}+\gamma_1\widetilde{\phi}_{Rd(k)}}$$
$$\left[i_{sq(k)}+\gamma_1\frac{T_d}{L_mT_a}\widetilde{\phi}_{Rq(k)}\right] \quad (26)$$
$$i_{sq(k+1)}=\frac{JL_R}{nL_m\Phi_{ref(k)}}(\frac{D}{J}w_{m(k)}-k_2\widetilde{w}_{m(k)}$$
$$+\frac{T_{l(k)}}{J}+\frac{w_{mref(k+1)}-w_{mref(k)}}{T_a}$$
$$-\gamma_1\left(k_2T_a-1\right)\widetilde{w}_{m(k)})-\frac{\gamma_1}{\Phi_{ref(k)}}$$
$$(i_{sq(k)}\widetilde{\phi}_{Rd(k)}-i_{sd(k)}\widetilde{\phi}_{Rq(k)}) \quad (27)$$

where a small γ_1 can guarantee condition (23) and a large k_2 provides well tuned responses in the tracking of $w_{m(k)}$. In section VI the exact design parameters values (k_2, γ_1 and δ) are chosen aiming accomplish conditions (21)-(23) and reach a controller with a suitable regulation performance.

Theorem 2. Considering the design input functions, equations (17)-(19), the candidate function, represented in expression (20), constitutes an ISS-Lyapunov function for the system (11)-(13) with respect to $\left(u_{w(k)},u_{sd(k)},u_{sq(k)}\right)$ *assuming smooth* $w_{sl(k)},i_{sd(k)}$ *and* $i_{sq(k)}$ *variables with bounded operation intervals.*

Proof. The state-to-input functions related to the system (11)-(13), $\sigma(\cdot)$ *in definition V.2, are determined by inverting the relation defined in (17)-(19) [9] and aplying singular values, more precisely,*

$$\widetilde{w}_{m(k)}=-\frac{1}{2}\frac{1}{\gamma_1\left(1-k_2T_a\right)}u_{w(k)}$$
$$+\frac{1}{2}\mu\frac{(i_{sq(k)}T_d-i_{sd(k)}w_{sl(k)}T_aT_R)T_aT_R}{\gamma_1(1-k_2T_a)(T_d^2+w_{sl}^2T_a^2T_R^2)}u_{sd(k)}$$
$$-\frac{1}{2}\mu\frac{(i_{sd(k)}T_d+i_{sq(k)}w_{sl(k)}T_aT_R)T_aT_R}{\gamma_1(1-k_2T_a)(T_d^2+w_{sl}^2T_a^2T_R^2)}u_{sq(k)}$$
$$\widetilde{\phi}_{Rd(k)}=-\frac{T_dT_aT_R}{\gamma_1(T_d^2+w_{sl}^2T_a^2T_R^2)}u_{sd(k)}$$
$$+\frac{w_{sl}T_a^2T_R^2}{\gamma_1(T_d^2+w_{sl}^2T_a^2T_R^2)}u_{sq(k)}$$
$$\widetilde{\phi}_{Rq(k)}=-\frac{w_{sl}T_a^2T_R^2}{\gamma_1(T_d^2+w_{sl}^2T_a^2T_R^2)}u_{sd(k)}$$
$$-\frac{T_dT_aT_R}{\gamma_1(T_d^2+w_{sl}^2T_a^2T_R^2)}u_{sq(k)}$$

where $T_R=\frac{L_R}{R_R}$ *and* $\mu=\frac{T_anL_m}{JR_R}$.

Then, expression (20) conform a ISS-Lyapunov function, i.e. σ *constitutes a* $\kappa-function$*, whenever matrix* $S\left(w_{sl(k)},i_{sd(k)},i_{sq(k)}\right)$ *presents constant and non-null gains. This property can be guaranteed assuming smooth* $w_{sl(k)},i_{sd(k)}$ *and* $i_{sq(k)}$ *variables with bounded operation intervals.* ■

Remark 5. It must be pointed out that direct and quadrature currents $(i_{sd(k)},i_{sq(k)})$ *presents constant dc values when field oriented control is achieved, moreover, since* $i_{sd(k)}$ *and* $i_{sq(k)}$ *constitutes the control inputs variables, they are limited in the following operation intervals*

$$-I_{sat}\leq i_{sd(k)}\leq I_{sat}$$
$$-I_{sat}\leq i_{sq(k)}\leq I_{sat}$$

where I_{sat} *represent the saturation value of the current variables (*$50A$ *in all simulations).*

In the same way, the slip frequency (w_{sl} *), due to construction characteristics of the induction motor, present constant small values (steady state) with the following operation interval*

$$0\leq w_{sl(k)}\leq w_{slmax}$$

where w_{slmax} *represent the maximum slip frequency value and depends of the machine's maximum load torque value (break down torque) [5].*

Finally, notice that with $i_{sd(k)}, i_{sq(k)}$ and $w_{sl(k)}$ inside the operation intervals, some smooth variations in their values does not have significant effect in the singular values of matrix S.

By theorems V.1 and V.2 it is possible to conclude that system (11)-(13) it is Input-to-State stable (ISS) with respect to variables $\left(u_{w(k)}, u_{sd(k)}, u_{sq(k)}\right)$.

V. Parametric Variations Considerations

The Input-to-State Stability (ISS) is a strong property for nonlinear systems. As in the continuous case, whenever this property can be assured for disturbances entering additively to the states in a stabilizing state discrete feedback law, estimates from a converging state observer can be used in a certainty equivalence approach [1].

Remark 6. It should be stressed that the design of the controller and the estimation algorithms objectify to validate the certainty equivalence approach used. Thus , simulations testing a robust behavior of the whole nonlinear system (controller and estimator) regard considerable estimation errors are realized. This robust performance could verify the connection between input-to-state characteristics and robust stability presented in [3].

In the reduced order model case of the IM, since T_L and R_R are unknown constants the expressions (8)-(10) are modified by replacing the variable parameters by their estimates $\widehat{T}_{L(k)}$ and $\widehat{R}_{R(k)}$. More precisely,

$$\begin{aligned} & w_{\phi(k)} = w_{m(k)} \\ & + \frac{\widehat{R}_{R(k)} L_m}{L_R \Phi_{ref(k)}} i_{sq(k)} - \frac{u_{sq(k)}}{\Phi_{ref(k)}} \end{aligned} \tag{28}$$

$$\begin{aligned} & i_{sd(k)} = \frac{L_R}{\widehat{R}_{R(k)} T_a L_m} (\phi_{ref(k)} - \phi_{ref(k-1)}) \\ & + \frac{\phi_{ref(k-1)}}{L_m} + \frac{L_R}{\widehat{R}_{R(k)} L_m} u_{sd(k)} \end{aligned} \tag{29}$$

$$\begin{aligned} & i_{sq(k+1)} = \frac{J L_R}{n L_m \Phi_{ref(k)}} (\frac{D}{J} w_{m(k)} \\ & - k_2 \widetilde{w}_{m(k)} + \frac{\widehat{T}_{L(k)}}{J} \\ & + \frac{w_{mref(k+1)} - w_{mref(k)}}{T_a} + \frac{u_{w(k)}}{T_a}) \end{aligned} \tag{30}$$

Introducing the parameter estimation errors

$$\widetilde{R}_{R(k)} = R_R - \widehat{R}_{R(k)} \tag{31}$$

$$\widetilde{T}_{L(k)} = T_L - \widehat{T}_{L(k)} \tag{32}$$

the regulation error dynamics, expressions (11)-(13), are recalculated as

$$\begin{aligned} \widetilde{w}_{m(k+1)} &= (1 - k_2 T_a) \widetilde{w}_{m(k)} \\ &+ \frac{T_a n L_m}{J L_R} (i_{sq(k)} \widetilde{\phi}_{rd(k)} - i_{sd(k)} \widetilde{\phi}_{rq(k)}) \\ &- \frac{T_a}{J} \widetilde{T}_{L(k)} + u_{w(k)} \end{aligned} \tag{33}$$

$$\begin{aligned} \widetilde{\phi}_{Rd(k+1)} &= \widetilde{\phi}_{Rd(k)} - \frac{TaR_R}{L_R} \widetilde{\phi}_{Rd(k)} \\ &+ T_a w_{sl(k)} \widetilde{\phi}_{Rq(k)} + \frac{T_a}{L_R} (L_m i_{sd(k)} \\ &- \Phi_{ref(k)}) \widetilde{R}_{R(k)} + T_a u_{sd(k)} \end{aligned} \tag{34}$$

$$\begin{aligned} \widetilde{\phi}_{Rq(k+1)} &= \widetilde{\phi}_{Rq(k)} - \frac{TaR_R}{L_R} \widetilde{\phi}_{Rq(k)} \\ &- T_a w_{sl(k)} \widetilde{\phi}_{Rd(k)} + \frac{T_a L_m}{L_R} i_{sq(k)} \widetilde{R}_{R(k)} \\ &+ T_a u_{sq(k)} \end{aligned} \tag{35}$$

It should be noted in equations (33)-(35), that the parameter estimation errors (considered as perturbations) are introduced additively to the states. Also, notice that setting $\left(\widetilde{R}_{R(k)}, \widetilde{T}_{L(k)}\right) = 0$, in expressions (33)-(35), equations (11)-(13) can be obtained.

In section IV some conditions that guarantees ISS property were presented. Hence, in the following section some typical simulations are presented considering this analysis and using the convergent observer proposed in [4].

VI. Simulations and Observability Analysis

In order to take advantage of the observability analysis of the reduced order IM model, presented in [6], it is assumed a constant known load torque (T_L) and a constant but unknown rotor resistance (R_R). Also, it is assumed the following conditions in all the simulations,

i) The discrete controller performance, using the convergent flux estimator proposed in [4], is investigated. Therefore in all simulations the estimated flux components are feedback.

ii) The observability conditions are satisfied, i.e., the motor speed and load torque do not present null values simultaneously.

iii) The ISS conditions are satisfied, i.e., the current saturation variables and load torque are given as $I_{sat} = 50A$ and $T_L = 50.4Nm \leq T_{Lmax}$.

iv) The observer initial conditions and design matrices, are given by

COO: $P_{(0)} = I_5$, $R_{k+1} = 2H_{k+1} P_{k+1/k} H_{k+1}^T + 10^{-3} I_2$ and $Q_k = 10^4 e_k^T e_k I_5 + 10^{-3} I_5$

ROO: $P_{(0)} = diag(2 \cdot 10^3, 2 \cdot 10^3, 1)$, $R_{k+1} = 8H_{k+1} P_{k+1/k} H_{k+1}^T + 10^{-2} I_2$ and $Q_k = 10^4 e_k^T e_k I_3 + 10^{-2} I_3$

The controller design parameters are given by $k_2 = 51$ and $\gamma_1 = 1e-9$.

In Fig. 1 we assume initial estimates close to the actual values, more precisely, $\Phi_{ref} = 1.13Wb$, $\Phi_{R(0)} = \widehat{\Phi}_{R(0)} = 0.0\,[Wb]$, $T_L = 50.4\,[Nm]$, $R_R = 0.706\Omega$ and $\widehat{R}_{R(0)} = 0.353\Omega$. In Fig. 2, the observer is initialized with unfavorable values, that is, initial conditions far from the actual ones, $\Phi_{ref} = 1.13Wb$, $\Phi_{R(0)} = 0.00$, $\widehat{\Phi}_{R(0)} = 5.65\,[Wb]$, $T_L = 50.4\,[Nm]$, $R_R = 0.706\Omega$ and $\widehat{R}_{R(0)} = 3.53\Omega$.

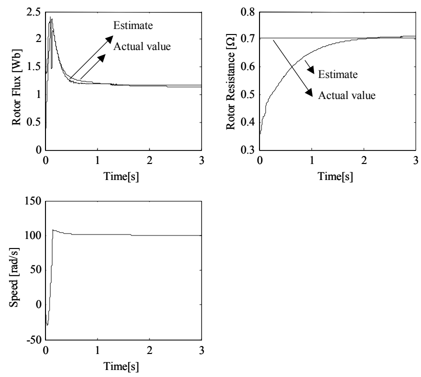

Fig. 1. Initial estimates close to actual values.

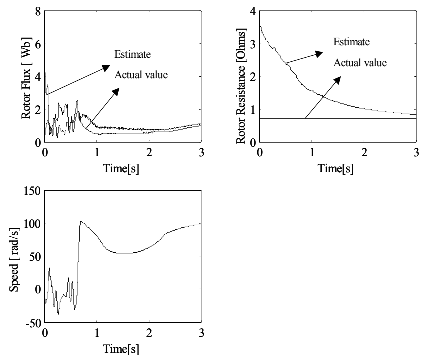

Fig. 2. Initial estimates far from actual values.

As figures 1 and 2 show, the estimation algorithm presents a suitable performance even with non favorable initializations. Both simulations represents the typical behaviour of the system, fact verified with several realizations. It should be noted in Fig. 2 that the certainty equivalence approach used is validated once the controller presents a robust behavior regard considerable estimation errors. This robust performance could verify the connection between input-to-state characteristics and robust stability presented in [3] considering the additively disturbances that estimation errors represents, equations (33)-(35).

VII. Conclusions

In this paper, an adaptive linearizing discrete controller for induction motors drives, using a reduced order model, is presented. The discrete time controller design is based on a continuous stabilization control technique proposed in [2] and the adaptive feedback control (observer based) using the discrete extensions of the input-to-state stability definitions [3].

Some stability considerations when assuming parametric variations were done in order to assure an input-to-state stability (ISS) proof of the proposed discrete controller. More precisely, assuming availability of states and non parametric variations, the convergence of the control system is determined by means of a Lyapunov analysis. Then, it is verified that the combination of an independently designed convergent observer and the asymptotic state-feedback controller assure stability for additive disturbances (parametric variations) [1], [9].

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