A DISCRETE VECTOR CONTROL OF SINUSOIDAL CURRENT PWM RECTIFIERS

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***Abstract*— Applications of PWM rectifiers are very promising, since they are able to supply DC power while keeping a sinusoidal current with unity in the fundamental power factor simultaneously. Another important application for PWM rectifiers is as static VAR compensators, where they supply reactive power to compensate inductive loads connected on grid. In this paper, a microprocessor-based regulator is proposed for voltage-fed PWM rectifiers with sinusoidal input current to control both the fundamental power factor and direct voltage. The grid current is controlled through a discrete-time feed-forward regulator and a state feedback control, which provide a fast response without steady-state errors. A outer DC voltage loop is composed by a PI regulator. Simulation results are presented for load variations and reactive power control.**

KEYWORDS

voltage source converter, rectifier, active power filter.

I. INTRODUCTION

The importance of AC/DC converters, as known as rectifiers, has increased continually because a large fraction of generated electric power is processed for use at the final DC load, including almost all applications others than motors, heaters, and lighting which are operated at the power-line frequency [1]. The conventional three-phase diode or thyristor rectifiers are used widely in many industrial applications where is required a high-power DC supply or an intermediate DC link of AC/AC converters. However, these rectifiers present serious operational problems:

- EMI interference;
- Decrease in the displacement power factor caused by excessive reactive power in AC side;
- AC power lines pollution due to a significant increase in levels of low-frequency current harmonics.

In recent years, three-phase switch-mode rectifiers with six switches have gained increasing interest among researchers. Each switch requires an individual driving circuit for its control, which become the control scheme more complicated. However, the switch-mode rectifier is a promising solution because the use of pulsewidth modulation technology (PWM) allows to obtain sinusoidal three-phase input currents. Another advantages of this structure are [2]:

- improvement of the supply current harmonic content in the presence of multiple non linear loads;
- improvement of the displacement power factor in presence of multiple loads with a leading or lagging power factor;
- improvement of the supply current balance;
- the power flow is bilateral, allowing a four-quadrant active rectifier operation.

There are two types of PWM rectifier: voltage-fed rectifier (VFR) and current-fed rectifier (CFR). The VFR is required to control the direct voltage while CFR is required to control the direct current. Both types can simultaneously to control the reactive power on grid, assuring a sinusoidal current with a desired power factor while supply DC power [3]. Thus, a PWM rectifier can operate as static VAR compensator, adjusting the power factor of any loads, filtering harmonic contents on power-lines and improving significantly the power quality on the power-distribution system.

The PWM rectifier is ideally applicable to DC-linked AC motor drives, since it draws sinusoidal input currents and controls the DC-bus voltage [4]. Considering a system with one or more motor drives, an active or passive rectifier can supplies

DC loads while performs the power quality compensation for nonlinear loads at the point of common coupling. However, the static VAR compensator requires an additional inverter and controller. The compensating current is regulated by an inner-loop current regulator, where the current references are computed from power theory using the measurement of both the nonlinear load currents.

Since the DC load varies, the robustness must be a very important characteristic for rectifier controllers. So far PI regulators have been used for rectifier control. However, the controlled variables are coupled with each other and the PI regulators design requires empirical knowlegde. Several methods for dq decoupling and control, similar to the field orientation (vector) control of AC machines, provide excellent performance both in voltage response and in low harmonic distortion [5]. In this approach, dq currents are controlled, providing a fast dynamic control and an excellent power factor. However, its implementation is quite complicated, requiring a significant computational charge and high-speed microprocessors or digital signal processors. An alternative is the use of state feedback control, which is essentially suited for systems with multi-input and multi-output, providing the following desirable features [6], [7]:

- guarantee of a stable system
- insensitivity to small parameter variations
- no state-state errors in response to a step change in the reference and/or disturbance

In this paper, a microprocessor-based regulator is proposed for voltage-fed PWM rectifiers with sinusoidal input current to control both the fundamental power factor and direct voltage. The grid current is controlled through a discrete-time feedforward regulator and a state feedback control, which provide a fast response without steady-state errors. A outer DC voltage loop is composed by a PI regulator. Simulation results are presented for load variations and reactive power control.

II. MATHEMATICAL MODEL OF VOLTAGE-FED RECTIFIER

Figure 1 shows a voltage-fed rectifier. Considering a balanced three-phase system, the AC side voltage equations can be expressed as

$$\frac{d\mathbf{i}_{abc}}{dt} = -\frac{R}{L}\mathbf{i}_{abc} + \frac{1}{L}\left(\mathbf{e}_{abc} - \mathbf{v}_{abc}\right) \qquad (1)$$

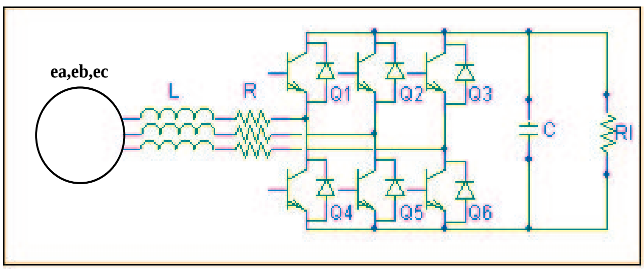

Fig. 1. PWM voltage-fed rectifier

To simplify the design and analysis, three-phase voltages and currents on the AC side can be converted into dq co-ordinate reference frames, which is rotating with the source angular frequency ω. This conversion can be performed in two stages. The first stage is the transformation of three-phase system abc to stationary two-axis system $\alpha\beta$, given by

$$\mathbf{x}_{\alpha\beta} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}\mathbf{x}_{abc} \qquad (2)$$

The second stage consists in the transformation of stationary two-phase system $\alpha\beta$ to rotating two-axis system dq

$$\mathbf{x}_{dq} = \frac{2}{3}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\mathbf{x}_{\alpha\beta} \qquad (3)$$

where $\theta = \omega t$. The conversion abc to dq co-ordinates could be executed directly too. However, as it will be seen later, the knowlegde of the variables in stationary co-ordinate $\alpha\beta$ is interesting to synchronize the currents references with the grid voltages. Thus, the voltage equation (1) are transformed to

$$\frac{d\mathbf{i}_{dq}}{dt} = \left[-\frac{R}{L}\mathbf{I} + \omega\mathbf{J}\right]\mathbf{i}_{dq} + \frac{1}{L}\left[\mathbf{e}_{dq} - \mathbf{v}_{dq}\right] \qquad (4)$$

Using the state transition matrix, an exact discrete-time version of the equation (4) for a sampling interval h is obtained directly as

$$\mathbf{i}_{dq}(k+1) = \mathbf{A_d}\mathbf{i}_{dq}(k) + \mathbf{B_d}\left[\mathbf{e}_{dq}(k) - \mathbf{v}_{dq}(k)\right] \qquad (5)$$

where:

$$\mathbf{A_d} = e^{-\frac{R}{L}h}\left[\mathbf{I}\cos(\omega h) + \mathbf{J}\sin(\omega h)\right] \qquad (6)$$

$$\mathbf{B_d} = \left[\mathbf{I} - \mathbf{A_d}\right]\frac{\mathbf{I}R + \mathbf{J}\omega L}{R^2 + (\omega L)^2} \qquad (7)$$

Assuming R_l as the DC load, the voltage equation on DC side will be

$$\frac{dV_{dc}}{dt} = \frac{\mathbf{v}'_{dq}\mathbf{i}_{dq}}{CV_{dc}} - \frac{V_{dc}}{CR_l} \qquad (8)$$

The AC voltages are related to DC voltage through the switching function. Although the rectifier input terminal voltage waveforms $\mathbf{v}_{dq}$ are pulse shaped, only the average component of the switching function is considered, since the harmonic components have little influence on the average power flow and can be neglected. Thus, the switching function is given by

$$\mathbf{S}_{dq} = \frac{\sqrt{2}\mathbf{v}_{dq}}{V_{dc}} \qquad (9)$$

The switching function $\mathbf{S}_{dq}$ corresponds to dq transformation of three-phase switching function $\mathbf{S}_{abc}$.

III. DISCRETE-TIME REGULATOR DESIGN

A. Grid current regulation

Defining the current error vector $\Delta\mathbf{i}_{dq}(k)$ to be minimized as

$$\Delta\mathbf{i}_{dq}(k) = \mathbf{i}_{dq}(k) - \mathbf{i}^*_{dq}(k) \qquad (10)$$

and the voltage error $\Delta\mathbf{v}_{dq}(k)$ caused by current error as

$$\Delta\mathbf{v}_{dq}(k) = \mathbf{v}^*_{dq}(k) - \mathbf{v}^f_{dq}(k) \qquad (11)$$

where $\mathbf{v}^{f}_{dq}(k)$ is a voltage value obtained from the current regulator feedforward. Considering that $\mathbf{B_d}$ is nonsingular, the expression for the feedforward regulator is obtained substituting the equations (10) and (11) into discrete-time model given by equation (5):

$$\mathbf{v}^{f}_{dq}(k) = \mathbf{B_d}^{-1}\left[\mathbf{A_d}\mathbf{i}^{*}_{dq}(k-1) - \mathbf{i}^{*}_{dq}(k)\right] + \mathbf{e}_{dq} \quad (12)$$

where a one-step delay is introduced to reference current since $\mathbf{i}^{*}_{dq}(k+1)$ is not available. Applying the state feedback

$$\Delta\mathbf{v}_{dq}(k) = \mathbf{G}\Delta\mathbf{i}_{dq}(k) \quad (13)$$

where $\mathbf{G}$ is the feedback gain 2×2 matrix. Since that

$$\Delta\mathbf{i}_{dq}(k+1) = [\mathbf{A_d} + \mathbf{B_d}\mathbf{G}]\,\Delta\mathbf{i}_{dq}(k) = \mathbf{H}\Delta\mathbf{i}_{dq}(k) \quad (14)$$

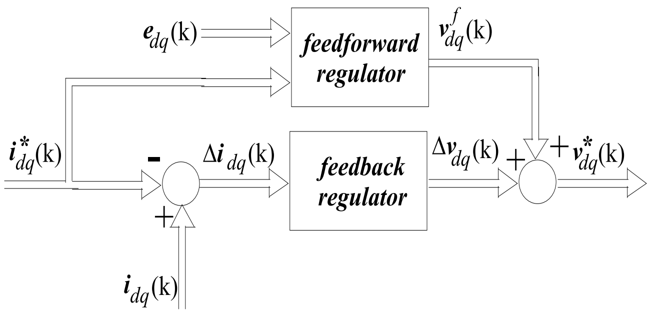

Fig. 2. Grid Current regulator

The gain matrix $\mathbf{G}$ must be determined so that $\mathbf{H}$ has its eigenvalues assigned inside the unit circle of the z-plane to decrease the current error. Thus, the gain matrix $\mathbf{G}$ can be determined as follows:

$$\mathbf{G} = \mathbf{B_d}^{-1}[\mathbf{H} - \mathbf{A_d}] \quad (15)$$

where the choice of $\mathbf{H}$ is made according to required error dynamics. The grid current regulator can be represented by the block diagram in Figure 2.

B. Synchronization to the grid

The grid currents are controlled in a rotating two-axis grid voltage orientated reference frame. An accurate voltage orientation for a grid connected converter becomes simple since the grid voltages are measured. For a symmetrical three-phase system, the trigonometrical functions of grid angle θ is given by

$$\cos\theta = \frac{e_\beta}{|\mathbf{e}|} \quad (16)$$

$$\sin\theta = \frac{e_\alpha}{|\mathbf{e}|} \quad (17)$$

so that $e_d = E_{rms}$ and $e_q = 0$. In this reference frame, the component i_d corresponds to active power while the component i_q represents the reactive power. Since i_d and i_q can be controlled independently, the reactive and active powers too. Thus, to obtain a sinusoidal current with unity in fundamental power factor, the reference i^*_q is matched to zero.

C. DC voltage regulation

To regulate the DC voltage, an input power must be injected to the capacitor to compensate the power delivered to the load. The control of the input power can be accomplished through the regulation of input current in coordenation with line voltage. Figure 3 shows the control block diagram for the DC voltage regulation of PWM rectifier.

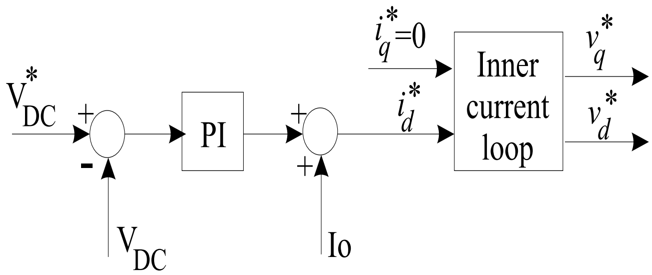

Fig. 3. DC voltage regulation

A PI controler is used for the voltage loop regulation. The reference current generated from the voltage loop is:

$$i^*_d = I_o + \left[K_P\Delta V_{dc} + K_I\sum\Delta V_{dc}\right] \quad (18)$$

where the compensation current I_o due to the load power requirement can be computed as

$$I_o = \sqrt{\frac{2}{3}}\frac{V_{dc}I_{dc}}{E_{rms}} \quad (19)$$

IV. PWM CONTROL OF VFR

Different ingenious methods for DC voltage control with nearly sinusoidal input currents are well known. Classical methods are based on hysteresis control of rectifiers legs following the sinusoidal reference or optimal PWM pattern for harmonic elimination. Basic tasks for pulsewidth modulation (PWM) controled rectifier are [6]:

- to provide a constant and adjustable DC link voltage with respect to the load changes and supply network imperfection;
- to ensure possibility of energy regeneration;
- to minimize line side harmonics injected by the rectifier switching;
- to provide unity power factor at the point of common coupling.

Due to its simplicity, this paper adopts the symmetrical regular PWM as pattern for rectifier modulation in despite of other modulation method. The width of command pulse for symmetrical regular PWM is given by [8]:

$$\tau_i(k) = \frac{T}{2}\left(1 + \frac{v_i(k)}{E}\right) \quad (20)$$

V. SIMULATION RESULTS

A computer simulation was done using MATLAB to verify the performance of proposed controller. The rectifier parameters and controller adjustment are given in table I. Considering

TABLE I

RECTIFIER PARAMETER AND CONTROLLER ADJUSTMENT

Grid	$125V_{rms}/60Hz$, $R = 0.3\Omega$, $L = 37mH$
D.C. link parameter	$C = 1100\mu F$
grid current controller	sampling interval $= 1ms$, $\mathbf{H} = .01\mathbf{I}$
PI DC Voltage controller	$K_P = 0.25$, $K_I = 0.12$

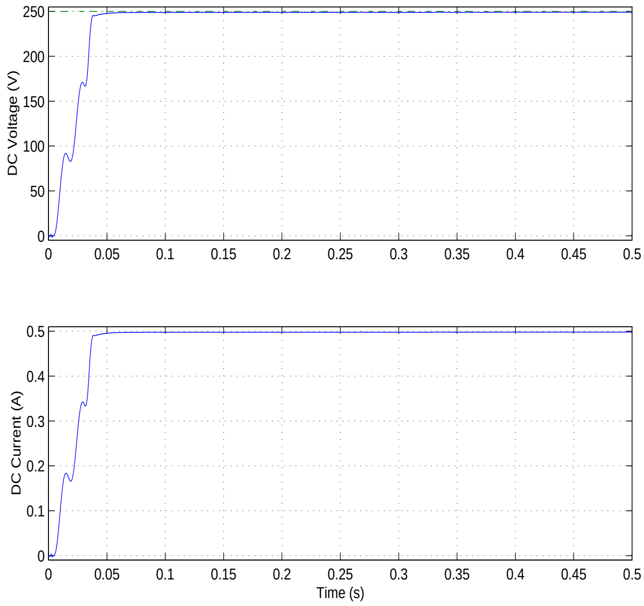

Fig. 4. DC voltage and current at start-up

$R_l = 500\Omega$ and DC voltage reference adjusted to 250 Vcc, the rectifier behavior during start-up is shown on figs. 4 and 5. Aiming to operate rectifier with unity in fundamental power factor, the reference of I_q component is matched to a null value. As shown in fig. 4, the proposed control system provides a satisfactory adjustment for DC voltage. Large currents variations arise on AC side during start-up as shown in fig 5, once the voltage $\mathbf{v}$ on bridge input is null until the establishment of an adequate switching by controller. Thus, only grid impedance limit the phase currents during start-up. At state-steady, the rectifier operates at unity power factor as shown the fig. 4.

The rectifier was submitted to a load variation. The initial $R_l = 500\Omega$ is changed suddenly to $R_l = 250\Omega$ and the rectifier behavior is shown in figs. 7 and 8. It is observed that direct voltage has a small drop and the rectifier remains operating at

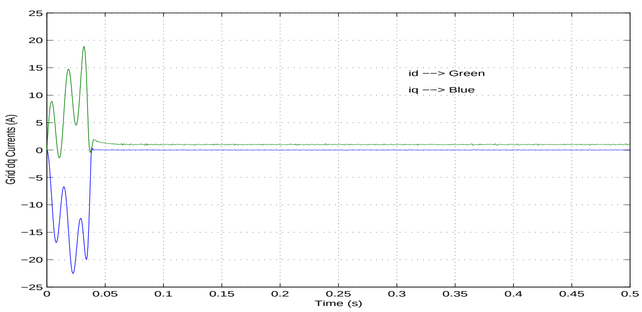

Fig. 5. dq grid currents at start-up

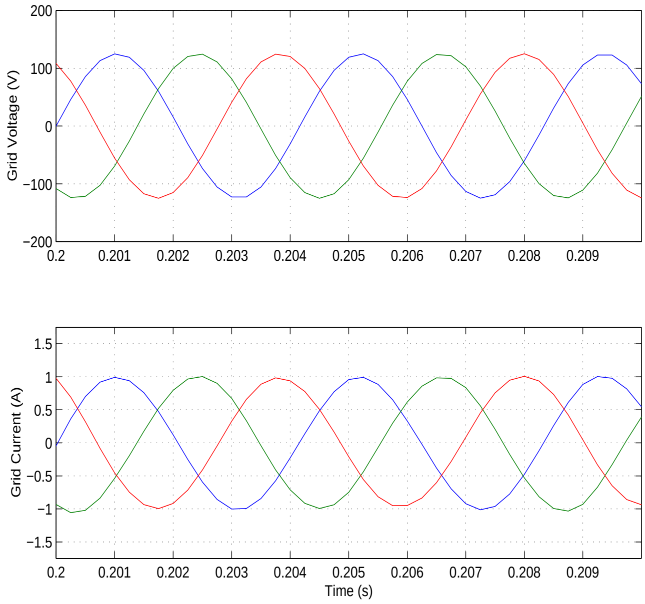

Fig. 6. Steady state three-phase grid voltages and currents

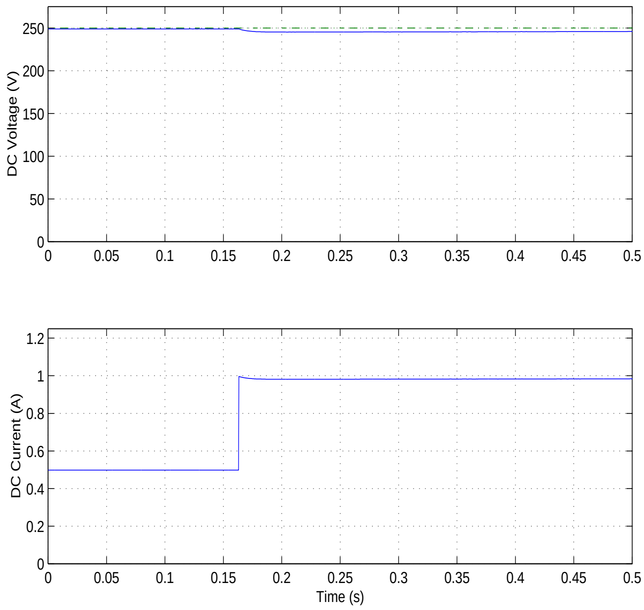

Fig. 7. DC voltage and current for load variation

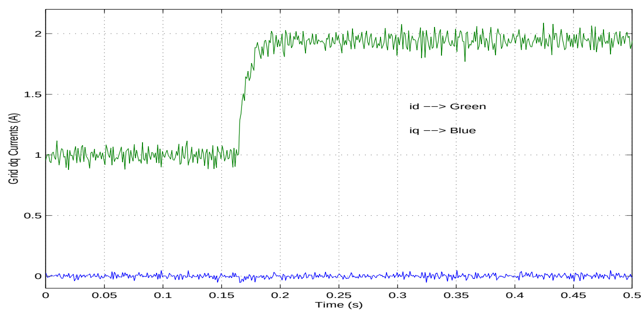

Fig. 8. dq Grid currents for load variation

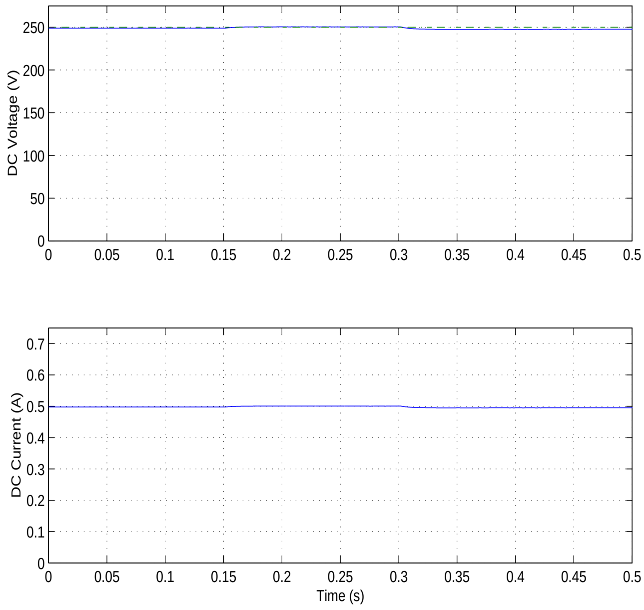

Fig. 9. DC voltage and Current for Iq variation

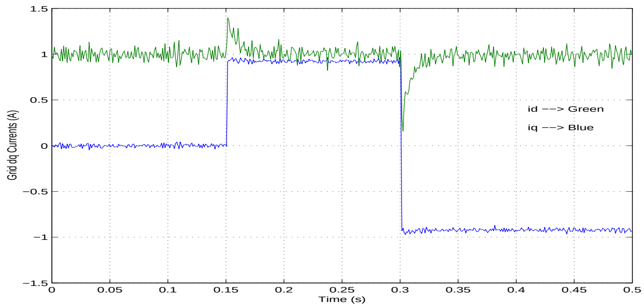

Fig. 10. *dq* Grid currents for Iq variation

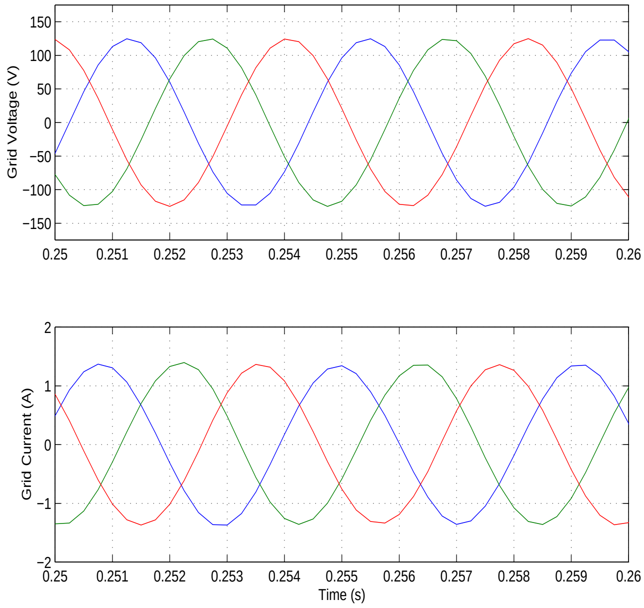

Fig. 11. Steady state three-phase grid voltages and currents at 0.25s

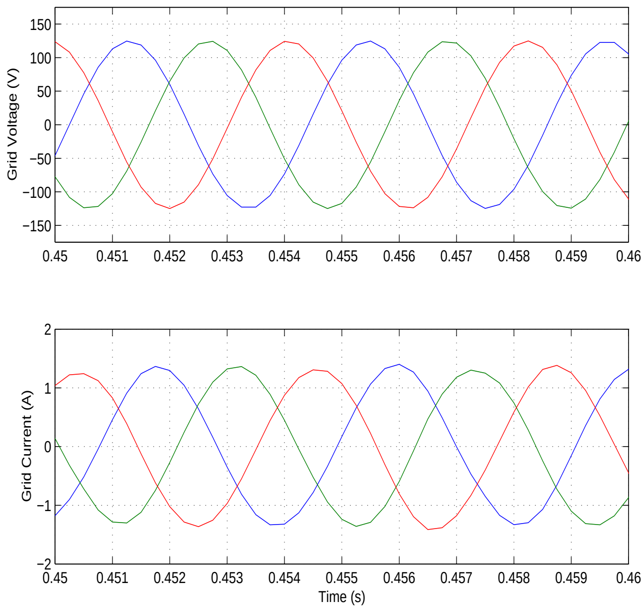

Fig. 12. Steady state three-phase grid voltages and currents at 0.45s

unitary power factor in spite of load variation. Obviously, the i_d component increases aiming to compensate the increase of load current.

A reactive power control is the last test of proposed control. The behavior of rectifier is shown in figs. 9 and 10. With rectifier operating at unity power factor ($i_q^* = 0$), a positive unitary step leaves i_q $+1A$. In this situation, the grid current advances in relation of grid voltage, as shown in fig 11, and the rectifier pass to actuate as an inductance connected to grid. After $0.3s$, the i_q reference is suddenly changed to $-1A$, and voltage grid advanced in relation to current grid. In this case, the rectifier acts as a capacitance connected to grid, as shown in fig. 12. It is noted that The influence of rective power control over DC voltage, DC current and i_d component is very small, although the variables is coupled.

VI. Conclusion

In this paper, a discrete-time controller is proposed for voltage-fed PWM rectifiers to adjust DC voltage and reactive power on grid. The grid current is controlled through a discrete-time feed-forward regulator and a state feedback control, which provide a fast response without steady-state errors. A outer DC voltage loop is composed by a PI regulator. The simulation results shown a good performance of proposed controller at start-up and during load variations, providing a good regulation of fundamental power factor and direct voltage.

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