THE COMPLEX CONTROLLER IN DIRECT TORQUE CONTROL STRATEGY FOR LOW SPEED INDUCTION MACHINE DRIVES USING THE COMPLEX TRANSFER FUNCTION CONCEPT.

Alfeu J. Sguarezi Filho, Edson Bim
Universidade Estadual de Campinas, FEEC
Cidade Universitária Zeferino Vaz
CP 6101 - 13083-970 - Campinas – SP
sguarezi@fee.dsce.unicamp.br,bim@fee.dsce.unicamp.br

Abstract: **A common induction machine problem is its control at the low speed. To solve this problem is applied the complex transfer function concept on induction machine model and it allows to design and tuning a complex controller by using frequency-response function for speed drives in a direct torque strategy. Another problem at low speed is the flux estimation. For this propose is used an integration method that permits the flux estimation at low speeds. Experimental results are presented for validation the controller.**

***Keywords*: Induction Motor, Mathematical Modeling, Direct Torque Control, Space State Approach, Space Phasor Modeling.**

I. INTRODUCTION

The dynamics of induction motor (IM) is represented traditionally by differentials equations. The space vector concept [1] is used in the mathematical representation of IM state variables like voltage, current and air gap flux.

The concept of complex transfer function is achieved by the application of Laplace transform to the differential equations in which the complex coefficients are in according spiral vector theory done by Yamamura [2].

Holtz [3] proposes an IM model using the complex transfer function and present simulation results. Others procedures of modeling and simulating of IM dynamics using the complex transfer function concept are presented by Cad[4].

Briz [5] applied the concept of transfer function to the design of current regulators for a RL load and to IM. The regulator design is realized by using the direct and quadrature axis.

In the same form of Briz, Holtz [6] designs a controller in complex form to stator current of IM. Simulation results showed that it allows work without cross-copplying in direct and quadrature currents.

An alternative for IM drives is the use of direct torque control (DTC) that consists in the direct control of stator flux λ_1 and torque T_e. DTC controller generates a stator voltage vector that permits quick torque response by the smallest variation of the stator flux.

Divan [7] proposed a DTC strategy using PI controllers and space vector modulation to generate a stator voltage based on torque and stator flux error. This strategy has good response of torque, although tests with low speed have not been shown yet.

This work proposes to substitute the conventional PI controller by a complex gain that is designed by using the complex transfer function concept and the tuning is realized by the frequency-response function of system controller-IM. Experimental results of low and high speeds are presented for validation the proposed controller. The DTC strategy is of Divan.

II. THE COMPLEX TRANSFER FUNCTION

The concept of complex transfer function is achieved by the application of Laplace transform to the differential equations in which the complex coefficients are in according spiral vector theory done by Yamamura [2].

A characteristic of the complex transfer functions is the possibility to produce the transient behavior of second real order system by means of first order complex system. Fig.1 presents a first order complex transfer function block diagram and the correspondent differential equation is gotten by:

$$\begin{gathered}\dot{\vec{y}}(t)+a\vec{y}(t)=\vec{u}(t)\\ \dot{\vec{y}}(t)+(\delta+j\omega)\vec{y}(t)=\vec{u}(t)\end{gathered} \tag{1}$$

From the Fig. 1 the unfolded real version of the complex transfer function represents a more complicated system. This occurs due to the cross coupling between the real and imaginary terms each variable.

III. COMPLEX MODEL OF THE INDUCTION MACHINE

The IM model is written in synchronous referential *(dq)* and the state variables are stator current $\vec{i}_1$ and stator flux $\vec{\lambda}_1$.

$$\begin{bmatrix}\vec{v}_1\\0\end{bmatrix}=\begin{bmatrix}R_1&0\\0&R_2\end{bmatrix}\begin{bmatrix}\vec{i}_1\\\vec{i}_2\end{bmatrix}+\begin{bmatrix}\dot{\vec{\lambda}}_1\\\dot{\vec{\lambda}}_2\end{bmatrix}+\begin{bmatrix}j\omega_1&0\\0&j(\omega_1-P\omega_{mec})\end{bmatrix}\begin{bmatrix}\vec{\lambda}_1\\\vec{\lambda}_2\end{bmatrix} \tag{2}$$

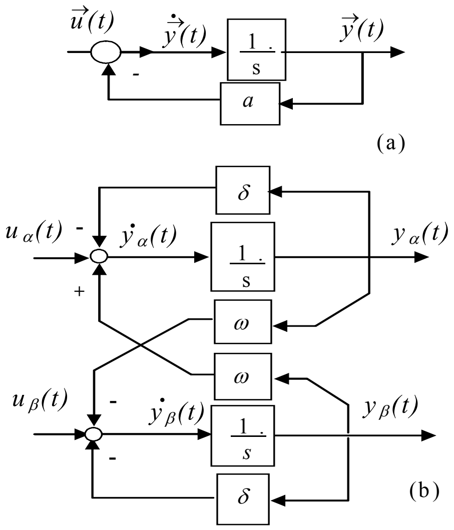

Fig.1. Block diagram of a complex transfer function and their equivalents: folded format (a) and unfolded format (b).

$$\begin{bmatrix}\vec{\lambda}_1\\ \vec{\lambda}_2\end{bmatrix}=\begin{bmatrix}L_1 & L_H\\ L_H & L_2\end{bmatrix}\begin{bmatrix}\vec{i}_1\\ \vec{i}_2\end{bmatrix} \tag{3}$$

$$Te=\frac{3}{2}P.\operatorname{Im}\left\{\vec{i}_1\vec{\lambda}_1^{*}\right\} \tag{4}$$

$$J\frac{d}{dt}\omega_{mec}=T_e-T_L \tag{5}$$

Where:

superscript * -represents the conjugate.
subscript *1* represents stator parameters.
subscript *2* represents rotor parameters.

$\vec{i}$ -current vector.
R -resistance.
$\vec{\lambda}$ - stator flux vector.
ω_1 -synchronous frequency of stator flux.
ω_{mec} -mechanical velocity.
σ -leakage coefficient.
L -inductance.
L_m -magnetization inductance.
P -number of pair of poles.
$\vec{v}$ -voltage vector.
J -load inertia.
T_L -load torque.

By combining (2) and (3), after some manipulations, it can be write as a complex space state equation:

$$\begin{bmatrix}\dot{\vec{\lambda}}_1\\ \dot{\vec{i}}_1\end{bmatrix}=\begin{bmatrix}a_1 & a_2\\ a_3 & a_4\end{bmatrix}\begin{bmatrix}\vec{\lambda}\\ \vec{i}_1\end{bmatrix}+\begin{bmatrix}\vec{v}_1\\ \dfrac{\vec{v}_1}{\sigma L_1}\end{bmatrix} \tag{6}$$

$$a_1=-j\omega_1 \tag{7}$$

$$a_2=-R_1 \tag{8}$$

$$a_3=\frac{R_2}{\sigma L_1 L_2}-j\frac{P\omega_{mec}}{\sigma L_1} \tag{9}$$

$$a_4=-\left[\left(\frac{R_1}{\sigma L_1}+\frac{R_2}{\sigma L_2}\right)+j\left(\omega_1-P\omega_{mec}\right)\right] \tag{10}$$

From now it will be assumed that the mechanical time constant is much greater then the electrical time constants. Thus, ω_{mec} constant is a valid approximation. By using the Laplace transform in (6) is obtained the complex transfer function of IM and its block diagram by using (4), (5) and (6) is shown in the Fig. 2.

IV. DIRECT TORQUE CONTROL

The use of direct torque control (DTC) permits a quick response of torque and consists the direct control of stator flux λ_1 and torque T_e. DTC controller generates a stator voltage vector that permits quick torque response by the smallest variation of the stator flux.

This work is based in Divan's strategy that uses a conventional PI controllers and space vector modulation to generate a stator voltage based on torque and stator flux error.

In this work, by using stator field orientation, the torque and stator flux must become parts of a complex number, where the magnitude of stator flux λ_1 is the real component and the torque T_e is the imaginary component. Hence, the reference signals and error signal become a complex number and controller is a complex gain *(a+jb);* this gain has the function of generating voltage reference vector by using the stator flux - torque vector error (ε_λ +j ε_T) and the stator voltage vector in the control strategy and it is given by

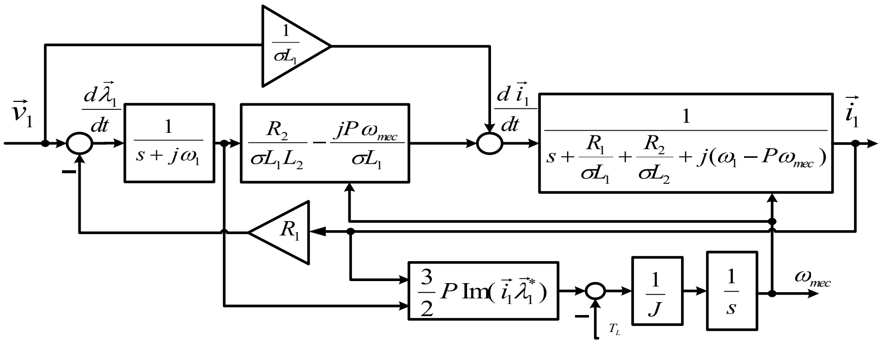

Fig. 2 Block diagram of induction machine with synchronous referential.

$$\vec{v}_1^* = (\varepsilon_\lambda + j\varepsilon_T)(a + jb) \quad (11)$$

what signifies that the direct and quadrature axis of the voltage vector are

$$v_d = \varepsilon_\lambda a - \varepsilon_T b \quad (12)$$

$$v_q = \varepsilon_\lambda b + \varepsilon_T a \quad (13)$$

Where:

a -real part of proportional complex gain.
b -imaginary part of proportional complex gain.
ε_λ -flux error signal.
ε_T -torque error signal.

The block diagram of model with the complex gain is presented in Fig. 3.

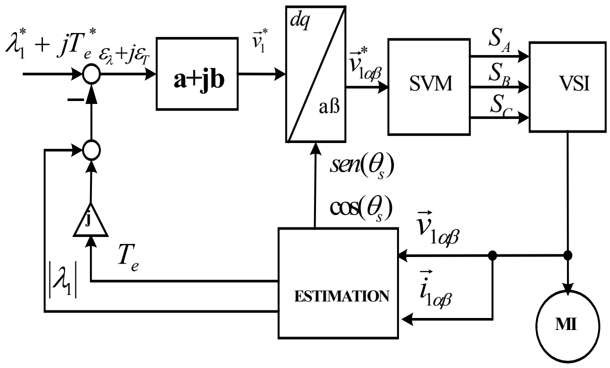

Fig. 3. The DTC strategy with complex controller.

It can be observed in (12) and (13) that the complex gain changes the amplitude and phase of the vector voltage.
The reference stator voltage vector is transformed by using stator flux position θ_s to obtain the stator voltage at stationary reference frame as done in the next section.

V. ESTIMATION BLOCK

The estimation of stator flux is calculated using the stator currents and voltages, given by

$$\vec{\lambda}_{1\alpha\beta} = \int \left(\vec{v}_{1\alpha\beta} - R_1 \vec{i}_{1\alpha\beta}\right) dt \quad (14)$$

Where:

subscript $\alpha\beta$ - stator stationary reference frame.

The stator flux angle is estimated by using the trigonometric transfer function

$$\theta_s = tg^{-1}\left(\frac{\lambda_{1\beta}}{\lambda_{1\alpha}}\right) \quad (15)$$

To allow good stator flux estimation by using a simple integrator, Hu [8] proposes an integration method based on

$$y = \frac{1}{s + \omega_c} x + \frac{\omega_c}{s + \omega_c} z \quad (16)$$

where:

x - input of integration.
z - compensation signal.
ω_c - cut frequency.

Assuming that the compensation signal is set to zero, the integrator becomes a first order LP filter that is usually adopted to replace the pure integrator in practice. If the compensation is taken from the integrator's output ($z=y$) the integrator performs the same function as a pure integrator $y = {}^{1}\!/_{s}\, x$. Another way to get a better pure integrator performance is the use of a limiter that is proposed by Hu. The integrator block diagram within limiter is shown in the Fig 4.

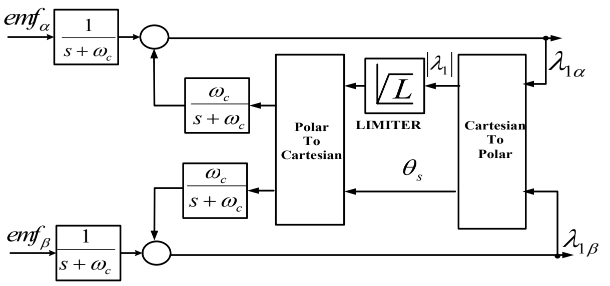

Fig. 4. Integrator block diagram.

VI. DESIGN OF COMPLEX GAIN

The tuning of complex gain requires the transfer function closed loop of system-IM, and then it can get frequency-response function. The IM closed loop transfer function, presented on Fig. 2, considering the slip approximated null, is given by

$$H(s)=\frac{\vec{i}_1}{\vec{v}_1}=\frac{\left(s+\dfrac{j\omega_1}{\sigma L_1}\right)+a_3}{\left(s+j\omega_1\right)\left(s+\dfrac{R_1}{\sigma L_1}+\dfrac{R_2}{\sigma L_2}\right)+R_1 a_3}. \quad (17)$$

To allow that *H(s)* (17) output be the stator flux magnitude λ_1 and torque T_e, it required a relationship by using the stator current vector $\vec{i}_1$. The relationship is given by

$$\lambda_1=\lambda_{1d}=\sigma L_1 i_{1d} \quad (18)$$

and the electromagnetic torque is calculated by

$$T_e=\frac{3}{2}P\lambda_1 i_{1q}. \quad (19)$$

The magnitude of stator flux is assumed to be essentially constant in (19). Thus, by using (17), (18) and (19) the new transfer function is

$$\frac{\lambda_1+T_e}{\vec{v}_1}=H(s)\left(\sigma L_1+jP\lambda_1\frac{3}{2}\right). \quad (20)$$

The FRF of (20) is presented in Figure 6 and it was chosen the *20 Hz* frequency to tuning the complex gain. Then, with (20) and (11) becomes possible to close the loop with the complex gain as Fig. 5 showed.

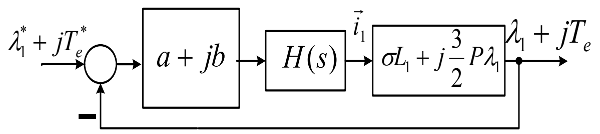

Fig. 5. System to design the complex gain.

The transfer function of Fig. 5 is calculated by

$$\frac{\lambda_1+T_e}{\lambda_1^*+T_e^*}=\frac{(a+jb)H(s)\left(\sigma L_1+jP\lambda_1\dfrac{3}{2}\right)}{1+(a+jb)H(s)\left(\sigma L_1+jP\lambda_1\dfrac{3}{2}\right)} \quad (21)$$

It was chosen a complex gain equal 125-j25. Thus, with this valor it becomes possible to calculate FRF of (21) and it can be seen in Fig. 7. Equation's (21) magnitude is near of *0dB*. Then, in this case the system is stable.

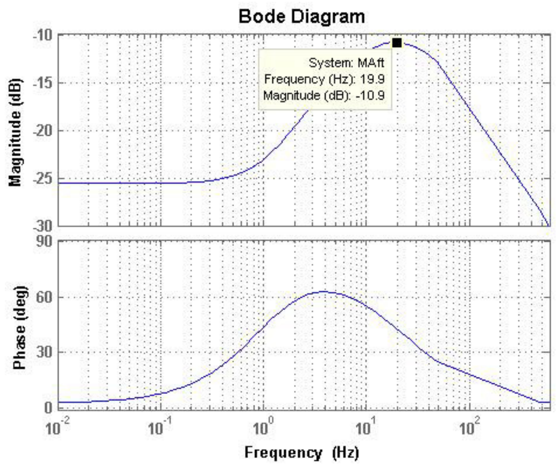

Fig. 6. Frequency-response function of (20).

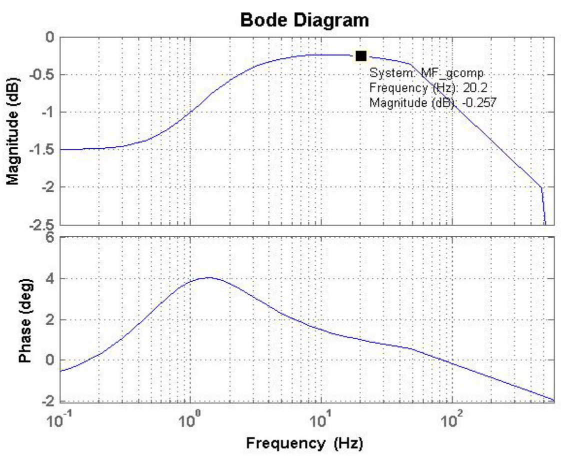

Fig. 7. Frequency-response function of (21).

VII. EXPERIMENTAL RESULTS

The DTC strategy was implemented using a Texas Instruments DSP TMS320F2812 platform. The system consists in a three-phase VSI with insulated-gate bipolar transistors (IGBTs). The stator voltage commands are modulated by using symmetrical space vector PWM, with constant switching frequency equal 2,5 *kHz*. The DC bus voltage of the VSI is 226 *V*. The stator voltages and currents are sampled with frequency equal 2,5 *kHz.* A conventional PI controller generates a torque reference by using the speed error. The motor parameters are shown at Appendix. The flux and torque estimation, and the flux-torque complex regulator and speed controller have the same sampling time equal a 2,5 *kHz*. The encoder resolution is 1500 points per revolution.

Five tests were made with no load IM. In the first respon-se to step torque operation of 12,2 *Nm* is shown in Fig. 8. It can be observed that the response time is 25ms and the reference is followed with oscillation. This occurs due to an error in process of measurement of the currents, the voltages.

The second test is a speed forward and reversal operation and it is presented in the Fig. 9. The speeds change of *600 rpm* to *-600 rpm* in 1.5s. It can be seen that the reference is followed in steady state and transient state.

Fig.10 presents a speed forward and reversal operation. The speeds change of *125 rpm* to *-125 rpm* in 1s. It can be seen the good performance of controller. The little bit error occurs due to the processes of measurement of speed.

The fourth test is speed forward and reversal operation as presented in the Fig. 11. The speeds change of *2 Hz/60 rpm* to 6 *Hz/180 rpm* in *300ms*. It can be seen the good performance of controller.

Fig. 12 presents a speed response to step operation. The speed changes of *60* to *-60 rpm*. It can be seen the good performance of controller.

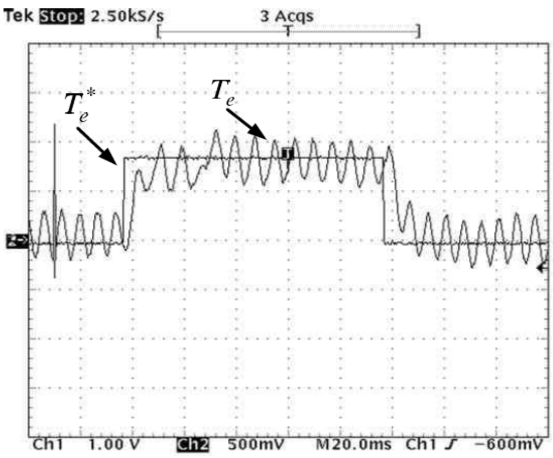

Fig 8. Response to step torque operation (9 Nm/div).

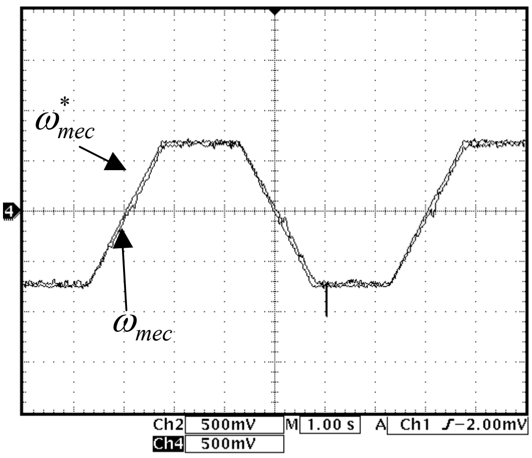

Fig. 9. Speed forward and reversal operation (500 rpm/div).

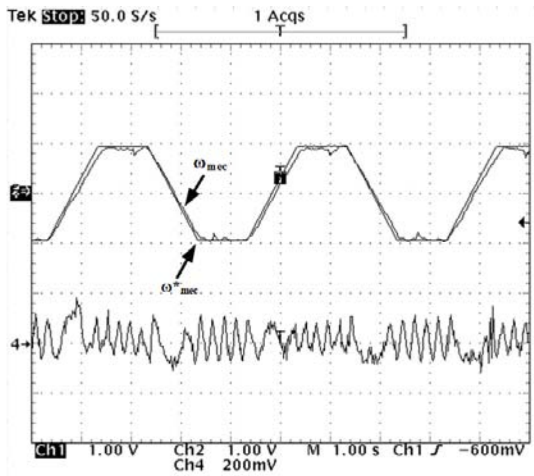

Fig. 10. Speed forward and reversal operation (125 rpm/div) and phase *a* current (6A/div).

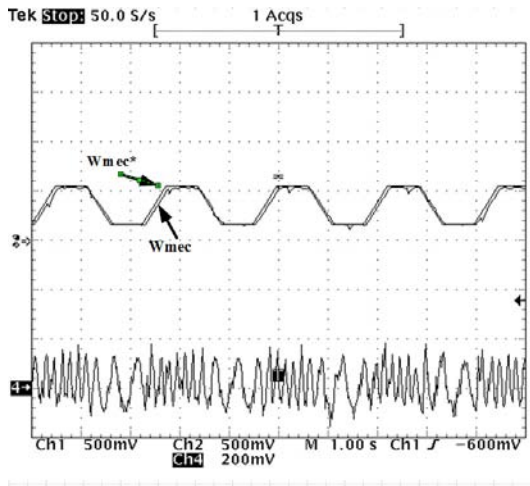

Fig. 11 Speed forward and reversal operation (150 rpm/div) and phase *a* current (6 A/div).

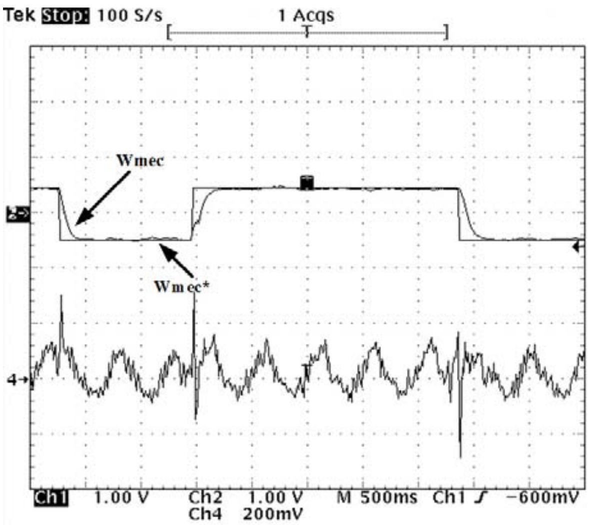

Fig. 12. Speed response to step operation (150 rpm/div) and phase a current (6 A/div).

VIII. CONCLUSION

The concept of complex vector notation associate with the complex gain allows the design and tuning the complex controller with the use FRF.

The experimental results presented that the control strategy have a satisfactory performance on transient and steady state although the complex gain design has been adjusted for a given speed, it works well for other speeds.

Thus, the complex vector becomes a good tool for implementation IM drives.

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APPENDIX

Machine parameters:

R1 = 2.229Ω, R2 = 1.522Ω ,L_m = 0.238485 H; L1 = 0.2470 H; L2 = 0.2497 H; J = 0.0067 kgm2; P = 2; PN = 2.3 kW, VN = 220 V .