STEADY-STATE ANALYSIS AND DESIGN METHODOLOGY FOR CLASS-E
RESONANT DC/DC CONVERTERS BASED ON A NORMALIZED
STATE-SPACE MODEL

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Abstract — This paper presents a steady-state analysis and design methodology for the Class-E² resonant DC/DC converter. The converter is represented in a normalized state-space model that is independent of specifications and real system components, like as inductors, capacitors and resistors. Analytical waveforms are obtained and can be used to design the converter. The main contribution of the paper is the development of an analysis methodology that does not separate the inverter and rectifier stages and ensures the soft-switching on both switch and diode. A step-by-step design procedure is presented, in which the real system components can be obtained based on design abacus. A comparison among theoretical, simulation and experimental results are performed based on a design example.

Keywords — Circuit Analysis, Resonant Power Conversion, State-Space Methods.

I. INTRODUCTION

By cascading a resonant DC/AC inverter to a resonant AC/DC rectifier, a resonant DC/DC converter is obtained [1]. This type of topology can be used as an electrical model for systems supplied by a DC input providing an AC output and concomitantly including an AC power conversion stage.

Resonant power converters are suitable when AC waveforms are required in applications like as wireless power transfer (WPT) [2]–[4], low power sensor nodes [5], [6], energy harvesting systems [7], [8] and so on. Usually, resonant DC/AC inverters can be used to supply piezoelectric transformers [9], [10], the primary side of WPT systems [11], [12] and ultrasonic transducers. Some resonant AC/DC rectifier applications can be founded for low power energy harvesting and lighting systems [13], [14]. In all instances, the motivation behind is to perform the soft-switching conditions: zero-voltage (ZVS), zero-current (ZCS) and zero-derivative (ZDS) switching, which make possible to reduce switching losses and increase the operating frequency [15]–[19].

Considering DC/DC resonant converters, Class-D, Class-E, Half/Fulle bridge with resonant tanks are commonly used for the inverter stage. Full-bridge topologies achieve higher transfer power ratio. On the other hand, the Class-E resonant inverter has the advantage that only one switch is used.

When the Class-E resonant inverter and the Class-E resonant rectifier are connected in series, the Class-E² resonant DC/DC converter is obtained as depicted in Figure 1, composed by: inductor \( L_f \), resonant tank \( L_r-C_r \), capacitor \( C_1 \), switch \( S \), capacitor \( C_2 \), diode \( D \), output filter \( L_f-C_f \) and load \( R_L \). The Class-E² converter operates at ZVS and ZDS conditions. When the switch \( S \) turns on, its voltage is zero; when the diode \( D \) turns off, its voltage is zero and the derivative of its voltage is zero. However, in order to achieve such optimal steady-state operation, it is necessary to obtain analytical waveforms, which have high mathematical complexity. Most of previous researches of the Class-E² resonant converter analyze the topology by separating the inverter and the rectifier stages [12], [15], [21]–[23]. This simplification can be used to design the converter, however, it is necessary to ensure the both stages are compatible with each other.

In this sense, this works present a steady-state analysis methodology, in which the analytical waveforms of the Class-E² resonant converter are obtained by considering the converter as only one combined circuit. It is specially challenging to solve the forth operating mode of the converter, in which the switch \( S \) and the diode \( D \) are off. It is because a circuit composed by four reactive elements \( C_1-L_r-C_r-C_2 \) is retrieved. For this purpose, a normalized state-space model is used to represent the converter, in which the system becomes independently of specifications like as input/output voltage, power and operating frequency; and real system parameters such as inductors, capacitors and resistors. By representing the converter by means of the resonant frequencies among the reactive components, it is possible to obtain the analytical waveforms ensuring the soft-switching conditions for any reasonable operating point. A step-by-step design procedure, in which by defining design specifications and selecting an operating point, the converter can be easily designed by means of normalized gain curves (design abacus) is presented as contribution. The proposed method is more suitable when it is necessary to design the converter without separating the inverter and the rectifier stages. The theoretical results are validate by means of simulation and experimental results.

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II. MATHEMATICAL APPROACH

The main features of the Class-\(E^2\) resonant DC/DC converter are the switch \(S\) zero-voltage switching and the diode \(D\) zero-derivative voltage switching. In this sense, the proposed modeling approach is going to be conducted by considering the following assumptions:

1. The input voltage source \(V_{in}\) and the input choke inductor \(L_i\) are replaced by a constant current source \(I_{in}\);
2. The output filter \(L_f-C_f\) and the load \(R_L\) are replaced by a constant current source \(I_o\);
3. The switch \(S\) and diode \(D\) off states are modeled as an open circuit;
4. The switch \(S\) and diode \(D\) on states are modeled as a short-circuit.

The model for the converter is portrayed in Figure 2:

\[
\begin{align*}
\frac{dv_{C_1}(t)}{dt} &= \frac{i_s(t)}{C_r}, \\
\frac{dv_{C_2}(t)}{dt} &= \frac{i_s(t)}{C_2}.
\end{align*}
\]

At mode II \((T_1 < t \leq D_1 2\pi)\): \(S\) on and \(D\) on. The following equations are considered:

\[
\begin{align*}
\frac{dv_{C_1}(t)}{dt} &= 0, \\
\frac{dv_{C_2}(t)}{dt} &= \frac{i_s(t)}{C_2}.
\end{align*}
\]

At mode III \((D_1 2\pi < t \leq T_2)\): \(S\) off and \(D\) on. The converter is governed by:

\[
\begin{align*}
\frac{dv_{C_1}(t)}{dt} &= \frac{i_s(t)}{C_1} - \frac{i_s(t)}{C_1}, \\
\frac{dv_{C_2}(t)}{dt} &= 0.
\end{align*}
\]

At mode IV \((T_2 < t \leq 2\pi)\): \(S\) off and \(D\) off. The differential equations are:

\[
\begin{align*}
\frac{dv_{C_1}(t)}{dt} &= \frac{i_s(t)}{L_r} - \frac{v_{C_2}(t)}{L_r}, \\
\frac{dv_{C_2}(t)}{dt} &= \frac{v_{C_2}(t)}{L_r} - \frac{v_{C_2}(t)}{L_r}.
\end{align*}
\]

The system is represented by a state-space model as follows, in which \(i\) is the index for the converter operating mode and it is described by: state matrix \(A_i\), input matrix \(B_i\), output matrix \(C_i\) and transmission matrix \(D_i\):

\[
x(t) = A_i x(t) + B_i u(t)
\]

and

\[
y(t) = C_i x(t) + D_i u(t),
\]

in which, the state vector \(x(t)\), output vector \(y(t)\) and input vector \(u(t)\) are written as:

\[
x(t) = [i_{s1}, v_{C_1}, v_{C_2}]^T,
\]

\[
y(t) = \begin{bmatrix}
th_i, v_{C_1}, v_{C_2}, i_s, i_p
\end{bmatrix}^T
\]

and

\[
u(t) = [I_{in}, I_o]^T.
\]
In (17), the variables are parameterized by the input current $I_i$ or the input voltage $V_{in}$. The input vector variables can be changed by the stored energy in the reactive components. In this sense, a new input vector $x_e(t)$ is selected as [20]:

$$
x_e(t) = \begin{bmatrix}
\sqrt{L_r}C_r & \sqrt{L_r}C_r & \sqrt{C_r} & \sqrt{C_r} & \sqrt{C_r} & \sqrt{C_r} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T,
$$

(19)

the new input vector is used to perform an equivalence transformation $P$ as follows:

$$
P = x_e(t) \cdot x(t)^{-1}.
$$

(20)

In (20), $x(t)$ and $x_e(t)$ must be evaluated in their diagonal matrices forms. In order to normalize the system in relation to the input current source $I_i$ and the operating angular frequency $\omega$, a normalization factor $\Gamma$ is defined as [20]

$$
\Gamma = \frac{\omega}{I_i} \sqrt{2C_2},
$$

(21)

the new state, input, output and transmission matrices, $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{C}_i$, and $\mathbf{D}_i$, respectively and considering the new vector space $x_e(t)$ are found by [20]:

$$
\mathbf{A}_i = \frac{\Gamma P \cdot A_i \cdot (\Gamma P)^{-1}}{\omega},
$$

(22)

$$
\mathbf{B}_i = \frac{\Gamma P \cdot B_i}{\omega},
$$

(23)

$$
\mathbf{C}_i = \frac{\Gamma P \cdot C_i \cdot (\Gamma P)^{-1}}{\omega},
$$

(24)

and

$$
\mathbf{D}_i = \mathbf{D}_i,
$$

(25)

which leads to:

$$
\mathbf{A}_i = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

$$
\mathbf{B}_i = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},
$$

$$
\mathbf{C}_i = \begin{bmatrix}
\frac{1}{\sqrt{L_rC_1}} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{L_rC_2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

$$
\mathbf{D}_i = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T.
$$


The terms in the state matrices can be rewritten as functions of unit-less parameters instead of real system parameters, like as $L_r$, $C_r$, $C_1$, and $C_2$ [24]. Towards this end, the resonant frequencies among the reactive components are used as

$$
\omega_1 = \frac{1}{\sqrt{L_rC_1}}; \quad \omega_2 = \frac{1}{\sqrt{L_rC_2}}; \quad \omega_3 = \frac{1}{\sqrt{L_rC_2}}.
$$

(26)

By normalizing the resonant frequencies by the operating angular frequency, unit-less parameters are achieved as

$$
A_1 = \frac{\omega_1}{\omega}; \quad A_2 = \frac{\omega_2}{\omega}; \quad A_3 = \frac{\omega_3}{\omega}.
$$

(27)

Parameters $A_1$, $A_2$ and $A_3$ are named as normalized resonant frequencies. Other unit-less parameters should be introduced. The quality factor $Q_L$ can be equated as function of the output load $R_L$ by [1]

$$
Q_L = \frac{R_L}{\omega_3 L_r} = \omega_1 C_2 R_L.
$$

(28)
The current transfer function $S$ is defined by the relation between the input and output currents as

$$S = \frac{I_o}{I_n}. \quad (29)$$

Furthermore, the input to output relation $a$ can be equated by [24]

$$a = \frac{V_n}{I_n R_L} \quad (30)$$

It can be seen that the terms in matrices $B_I, B_{II}, B_{III}, B_{IV}, D_I, D_{II}, D_{III}$ and $D_{IV}$ can be re-positioned into the input vector $u(t)$ by considering

$$u(\omega t) = \left[ I_{in} \ \ I_0 \ \ I_{in} \right]^T = \left[ \frac{1}{1 + \alpha} \right]^T.$$

By using (26), (27), (28), (29) and (30), the terms in the state matrices can be replaced by unit-less parameters and a normalized state-space model can be used as mathematical model for the converter as [25]

$$x_e(\omega t) = E_{[A_1, A_2, A_3]} x_e(\omega t) + F_{[A_1, A_3]} u(\omega t) \quad (32)$$

and

$$y(\omega t) = G_{[A_1, A_2, A_3]} x_e(\omega t) + H_u u(\omega t). \quad (33)$$

$E_i, F_i, G_i$ and $H_i$ are the unit-less state matrices, in which

$E_I = \begin{bmatrix} 0 & -A_2 & 0 & -A_3 \\ A_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_3 & 0 & 0 & 0 \end{bmatrix}, \quad F_I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix},$

$E_{II} = \begin{bmatrix} 0 & -A_3 & 0 & 0 \\ A_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -A_1 & 0 & 0 & 0 \end{bmatrix}, \quad F_{II} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$

$E_{III} = \begin{bmatrix} 0 & -A_2 & A_1 & 0 \\ A_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -A_1 & 0 & 0 & 0 \end{bmatrix}, \quad F_{III} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$

$E_{IV} = \begin{bmatrix} 0 & -A_3 & A_1 & 0 \\ A_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -A_1 & 0 & 0 & 0 \end{bmatrix}, \quad F_{IV} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$

$G_I = \begin{bmatrix} A_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_3 & 0 & 0 & 0 \end{bmatrix}, \quad H_I = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$

$G_{II} = \begin{bmatrix} A_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_3 & 0 & 0 & 0 \end{bmatrix}, \quad H_{II} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$

$G_{III} = \begin{bmatrix} A_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_3 & 0 & 0 & 0 \end{bmatrix}, \quad H_{III} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$

$G_{IV} = \begin{bmatrix} A_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_3 & 0 & 0 & 0 \end{bmatrix}, \quad H_{IV} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

and $H_{IV} = 0.$

By symbolically solving the state-space representation for each operating mode ($i = I, II, III, IV$), steady-state solutions can be achieved. An algorithm is proposed to solve the system, depicted as a flowchart in Figure 4. For the flowchart, the dotted lines represent the outputs and the continuous lines the changes of states defined by the operation related to $S$ and $D.$

![Fig. 4. Flowchart to solve the Class-E^2 resonant DC/DC converter model.](image-url)

In order to solve mode $I,$ symbolical initial conditions are used as: initial resonant inductor current $i_{L}(0),$ initial resonant capacitor voltage $v_{C}(0),$ initial capacitor $C_1$ voltage $v_{C_1}(0)$ and initial rectifier capacitor $C_2$ voltage $v_{C_2}(0).$ By solving mode $I,$ one equation for each state variable is found, which are dependent of the unit-less parameters and as function of $\omega t.$ By replacing $\omega t$ by $T_1,$ which represents the end of mode $I,$ the initial conditions to solve mode $II$ are achieved and can be described by $i_{L}(T_1),$ $v_{C}(T_1),$ $v_{C_1}(T_1)$ and $v_{C_2}(T_1).$ By solving mode $II$ and replacing $\omega t$ by $D_{II},$ which characterizes the end of mode $II$, the initial conditions for mode $III$ are set as $i_{L}(D_{II}, 2\pi), v_{C}(D_{II}, 2\pi), v_{C_1}(D_{II}, 2\pi)$ and $v_{C_2}(D_{II}, 2\pi).$ The same concept is used for mode $III,$ which ends at $T_2,$ being $i_{L}(T_2),$ $v_{C}(T_2),$ $v_{C_1}(T_2)$ and $v_{C_2}(T_2)$ the initial conditions to solve mode $IV.$ The last operating mode ends at $2\pi,$ which leads to the final conditions described as $i_{L}(2\pi), v_{C}(2\pi), v_{C_1}(2\pi)$ and $v_{C_2}(2\pi).$ The aforementioned symbolical solutions are dependent of nine parameters $A_1, A_2, A_3, a, D_c, i_{L}(0), v_{C}(0),$ $v_{C_1}(0)$ and $v_{C_2}(0).$
By equating the final conditions as equal to the initial conditions (steady-state condition), a linear system composed by four equations can be evaluated. In order to ensure the soft-switching operation of both switch $S$ and diode $D$, the zero-voltage switching conditions must be addressed to the linear system. When the switch $S$ turns on at $2\pi$, the capacitor $C_1$ voltage is 0 and the input current $I_{in}$ is equal to the resonant current $i_L$. When the diode turns off at $T_3$, the capacitor $C_2$ voltage and its derivative are 0. The following steady-state and soft-switching conditions are considered:

- $i_L(2\pi) = i_L(0)$ (steady-state condition);
- $v_{C_1}(2\pi) = v_{C_1}(0)$ (steady-state condition);
- $v_{C_2}(2\pi) = 0$ (switch $S$ ZVS condition);
- $i_L(T_3) = I_{in}$ (diode $D$ ZVS condition);
- $v_{C_1}(T_3) = 0$ (diode $D$ ZVS condition);
- $v_{C_2}(T_3) = I_{in}$ (diode $D$ ZVS condition).

The inherent system condition for capacitor $C_1$ voltage simplifies the linear system by equating $v_{C_1}(0) = 0$. Based on that, the following linear system is equated:

$$
\begin{bmatrix}
    i_L(2\pi) \left|_{A_1 A_2 A_3 A_4} \right. & i_L(0) & v_{C_1}(0) & v_{C_2}(0) & v_{C_3}(0) & v_{C_4}(0) & D_1 & T_1 & T_2 & a \\
    v_{C_1}(2\pi) \left|_{A_1 A_2 A_3 A_4} \right. & v_{C_1}(0) & 0 & v_{C_2}(0) & v_{C_3}(0) & v_{C_4}(0) & D_1 & T_1 & T_2 & a \\
    v_{C_2}(2\pi) \left|_{A_1 A_2 A_3 A_4} \right. & v_{C_2}(0) & v_{C_2}(0) & 0 & v_{C_3}(0) & v_{C_4}(0) & D_1 & T_1 & T_2 & a \\
    i_L(T_3) \left|_{A_1 A_2 A_3 A_4} \right. & i_L(T_3) \left|_{A_1 A_2 A_3 A_4} \right. & v_{C_1}(0) & v_{C_2}(0) & v_{C_3}(0) & v_{C_4}(0) & D_1 & T_1 & T_2 & a \\
\end{bmatrix} =
\begin{bmatrix}
    i_L(0) \\
    v_{C_1}(0) \\
    v_{C_2}(0) \\
    0 \\
\end{bmatrix} =
\begin{bmatrix}
    \frac{1}{a} \sqrt{c} \\
    \frac{1}{a} \sqrt{c} \\
    \frac{1}{a} \sqrt{c} \\
\end{bmatrix}
$$

(34)

Note that the diode turn-on and turn off are consequences of the system operation and cannot be controlled. The system has 7 equations and 10 variables. In this case, a parameter of the system operation and cannot be controlled. The system can be solved. By solving the output equation, the normalized steady-state values on the switches can be calculated and normalized gain and component stress curves can be obtained. Furthermore, by solving the output equation, the normalized steady-state waveforms are obtained as shown in Figure 5. The waveforms considering duty cycle, 0.4, 0.5, 0.6 and 0.7 are depicted in Figures 5(a)-(i), Figures 5(g)-(l), Figures 5(m)-(r) and Figures 5(s)-(x), respectively. It can be seen that the switch $S$ ZVS condition is ensured for all considered values of $D_1$ as shown in Figures 5(c),(i),(o),(u). The diode $D$ ZDVS condition is achieved as depicted in Figures 5(d),(j),(p),(v).

The main normalized and component stress curves are shown in Figure 6.

### III. DESIGN METHODOLOGY

In this section, a design methodology is proposed for the Class-E$^2$ converter. A step-by-step procedure is going to be performed.

**A. Step 1: Select the operating point**

The normalized gain and component stress curves depicted in Figure 6, represent the behavior of the converter. For instance, if the application requires a higher current gain, it can be necessary to use duty cycle less than 0.5, as shown in Figure 6(a). However, the lower the duty cycle, the greater the component stress, as portrayed in Figure 6(e)-(h).

Aiming to clarify the design methodology, some normalized results depicted in Figure 6 are shown in Table I.

#### TABLE I

<table>
<thead>
<tr>
<th>Acceleration</th>
<th>$A_2$</th>
<th>$A_1$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$I_{in}$</th>
<th>$Q_L$</th>
<th>$S$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>0.059</td>
<td>4.319</td>
<td>0.724</td>
<td>0.069</td>
<td>4.154</td>
<td>0.057</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.35</td>
<td>0.222</td>
<td>4.393</td>
<td>0.758</td>
<td>0.086</td>
<td>3.147</td>
<td>0.100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.375</td>
<td>4.467</td>
<td>0.804</td>
<td>0.106</td>
<td>2.441</td>
<td>0.167</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.45</td>
<td>0.520</td>
<td>4.542</td>
<td>0.865</td>
<td>0.128</td>
<td>1.933</td>
<td>0.267</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.659</td>
<td>4.620</td>
<td>0.946</td>
<td>0.152</td>
<td>1.555</td>
<td>0.412</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.55</td>
<td>0.791</td>
<td>4.700</td>
<td>1.055</td>
<td>0.179</td>
<td>1.272</td>
<td>0.617</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.60</td>
<td>0.920</td>
<td>4.783</td>
<td>1.203</td>
<td>0.210</td>
<td>1.050</td>
<td>0.906</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.65</td>
<td>1.045</td>
<td>4.868</td>
<td>1.406</td>
<td>0.247</td>
<td>0.872</td>
<td>1.314</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.70</td>
<td>1.166</td>
<td>4.955</td>
<td>1.698</td>
<td>0.291</td>
<td>0.724</td>
<td>1.903</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>1.284</td>
<td>5.035</td>
<td>2.135</td>
<td>0.347</td>
<td>0.599</td>
<td>2.784</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>1.397</td>
<td>5.042</td>
<td>2.854</td>
<td>0.424</td>
<td>0.490</td>
<td>4.153</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**B. Step 2: Define specifications**

The second step to design the converter based on the previously theoretical results is to define specifications, such as: output voltage $V_o$, operating frequency $f$ and output power $P_o$. Based on that, the operating angular frequency $\omega$, the output current $I_o$, the input current $I_{in}$, input voltage $V_{in}$ and the load $R_L$ are calculated by: $\omega = 2\pi f$, $I_o = P_o/V_o$, $I_{in} = I_o/S$, $V_{in} = P_o/I_{in}$ and $R_L = V_o/I_o$.

**C. Step 3: Design of the resonant circuit**

The designing equations for resonant circuit composed by $C_1$, $L_r$, $C_r$ and $C_2$ can be derived from (26), (27) and (28). The
The output filter inductor $L_f$ can be designed in the same way as $L_c$, by considering a normalized resonant frequency $A_f$ between $L_f$ and $C_f$:

$$A_f = \frac{\omega_f}{\omega}$$  \hspace{1cm} (44)

being $\omega_f = 1/\sqrt{L_fC_f}$. The equation for $L_f$ is dependent of $T_1$ and $T_2$:

$$L_f = \frac{(1-T_2+T_1)R_L}{A_f\omega}$$  \hspace{1cm} (45)

Finally, the output filter capacitor $C_f$ is designed as [1]

$$C_f = \frac{25}{\omega^2 L_f}$$  \hspace{1cm} (46)

### IV. RESULTS

This section shows a comparison among theoretical, simulation and experimental results. Let us consider a step-down low-power application that requires an output voltage of $4\,V$, output power of $800\,mW$ and operating frequency of $800\,kHz$. It can be seen in the design abacus in Figure 6(a) and Figures 6(e)-(h) that for duty cycle 0.5, a reasonable current transfer function and component stresses are achieved. By selecting $D_c = 0.5$, the unit-less parameters are extracted from

\[
\begin{align*}
D_c &= 0.4 & V_{C_1} & \frac{I_{L_1}}{I_{T_{in}}} \\
D_c &= 0.5 & V_{C_2} & \frac{I_{L_1}}{I_{T_{in}}} \\
D_c &= 0.6 & V_{C_1} & \frac{I_{L_1}}{I_{T_{in}}} \\
D_c &= 0.7 & V_{C_1} & \frac{I_{L_1}}{I_{T_{in}}} \\
\end{align*}
\]

Fig. 5. Normalized waveforms for the Class-E^2 resonant converter considering $A_2 = 0.75$ and $A_3 = 1.25$. (a)-(f) $D_c = 0.4$. (g)-(l) $D_c = 0.5$. (m)-(r) $D_c = 0.6$. (s)-(x) $D_c = 0.7$.}

The following equations are used:

\[
\begin{align*}
C_1 &= \frac{A_3 Q_l}{A_1^2 R_L \omega} \\
L_r &= \frac{R_L}{A_3 Q_l \omega} \\
C_r &= \frac{A_3 Q_l}{A_2^2 R_L \omega} \\
C_2 &= \frac{Q_l}{A_3 R_L \omega} \\
\end{align*}
\]

**D. Step 4: Design of the choke inductor and output filter**

The choke inductor $L_c$ can be designed by defining a normalized resonant frequency $A_c$ between $L_c$ and $C_1$ as follows:

$$A_c = \frac{\omega_c}{\omega}$$  \hspace{1cm} (42)

in which, the angular frequency is $\omega_c = 1/\sqrt{L_cC_1}$. The lower $A_c$, the lower the ripple in the choke inductor $L_c$, which is calculated as

$$L_c = \frac{\lambda_1^2 R_L}{A_c Q_l \omega}$$  \hspace{1cm} (43)
the design abacus and are described in Table I. The output current and the required input current are calculated in Step 2 of Section III. The converter components are designed by (38), (39), (40), (41), (43), (45) and (46). The specifications, selected operating point and designed components are shown in Table III. The converter was assembled for the experimental results. The results are shown in Figure 7. The simulation results were obtained from SPICE software and the theoretical results from a mathematical software. The soft-switching conditions were achieved. A quantitative comparison among theoretical, simulation and experimental results is going to be performed in order to analyze the error. The error analysis is shown in Table II. For the analysis the choke inductor and the output filter were replaced by constant current sources. This assumption leads to a restriction on the method, which is related to the possible values for $A_c$ and $A_f$. These values can not be close to $A_1$, $A_2$ and $A_3$ in order to ensure that $L_c$, $L_f$ and $C_f$ do not affect the behavior of the resonant circuit.

### TABLE II

<table>
<thead>
<tr>
<th>Error Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak value</td>
</tr>
<tr>
<td>$I_{Dpeak}$</td>
</tr>
<tr>
<td>$V_{Dpeak}$</td>
</tr>
<tr>
<td>$V_{Dpeak}$</td>
</tr>
<tr>
<td>$V_o$</td>
</tr>
</tbody>
</table>

### TABLE III

<table>
<thead>
<tr>
<th>Specifications / Components</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duty cycle, $D_c$</td>
<td>0.5</td>
</tr>
<tr>
<td>Operating frequency, $f$</td>
<td>800kHz</td>
</tr>
<tr>
<td>Output voltage, $V_o$</td>
<td>4V</td>
</tr>
<tr>
<td>Output power, $P_o$</td>
<td>800mW</td>
</tr>
<tr>
<td>Output current, $I_o$</td>
<td>200mA</td>
</tr>
<tr>
<td>Input current, $I_{in}$</td>
<td>128mA</td>
</tr>
<tr>
<td>Current transfer factor, $S$</td>
<td>1.557</td>
</tr>
<tr>
<td>Normalized resonant frequency, $A_1$</td>
<td>0.94</td>
</tr>
<tr>
<td>Normalized resonant frequency, $A_2$</td>
<td>0.75</td>
</tr>
<tr>
<td>Normalized resonant frequency, $A_3$</td>
<td>1.28</td>
</tr>
<tr>
<td>Choke inductor ripple as a normalized resonant frequency, $A_r$</td>
<td>0.09</td>
</tr>
<tr>
<td>Output ripple as a normalized resonant frequency, $A_f$</td>
<td>0.1</td>
</tr>
<tr>
<td>Diode turn-on time, $t_1$</td>
<td>0.659</td>
</tr>
<tr>
<td>Diode turn-off time, $t_2$</td>
<td>4.620</td>
</tr>
<tr>
<td>Resonant inductor, $L_r$</td>
<td>22μH</td>
</tr>
<tr>
<td>Resonant capacitor, $C_r$</td>
<td>3.3mF</td>
</tr>
<tr>
<td>Capacitor, $C_1$</td>
<td>2.5mF</td>
</tr>
<tr>
<td>Capacitor, $C_2$</td>
<td>1.2mF</td>
</tr>
<tr>
<td>Choke inductor, $L_c$</td>
<td>260μH</td>
</tr>
<tr>
<td>Output filter inductor, $L_f$</td>
<td>175μH</td>
</tr>
<tr>
<td>Output filter capacitor, $C_f$</td>
<td>25nF</td>
</tr>
<tr>
<td>Load, $R_L$</td>
<td>20Ω</td>
</tr>
</tbody>
</table>
Fig. 7. Results. (a)-(c) Inductor $L_r$ current. (d)-(f) Switch $S$ voltage. (g)-(i) Capacitor $C_2$ voltage. (j)-(l) Output voltage.
V. CONCLUSION

A steady-state analysis and design methodology for Class-E\(^2\) resonant DC/DC converters was presented in this work. This was the first state-space representation for such combination of a Class-E inverter and a Class-E rectifier. The proposed model is independent of specifications and real system parameters, which allows to obtain analytical waveforms and normalized gain and component stress curves. The state-space model allows to solve all operating modes. Subsequently, a linear system that includes steady-state and soft-switching conditions can be evaluated. The Class-E\(^2\) converter was assembled based on a proposed design methodology. The method can be used to analyze resonant DC/DC converters without separating the inverter to the rectifier stage. In contrast, the proposed method is more complex than other works. In this sense, it is concluded that a trade-off between complexity and accuracy should be considered. In this sense, this section shows the normalized state-space model is described by the following matrices:

\[
E_I = \begin{bmatrix} 0 & -A_2 & 0 & -A_1 \\ A_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -A_3 & 0 & 0 & 0 \end{bmatrix}, \quad F_I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix},
\]

\[
E_{II} = \begin{bmatrix} 0 & -A_2 & 0 & 0 \\ A_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -A_1 & 0 & 0 & 0 \end{bmatrix}, \quad F_{II} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]

\[
E_{III} = \begin{bmatrix} 0 & -A_2 & A_1 & -A_3 \\ -A_3 & 0 & 0 & 0 \\ -A_2 & 0 & 0 & 0 \\ -A_1 & 0 & 0 & 0 \end{bmatrix}, \quad F_{III} = \begin{bmatrix} 0 & 0 & 0 \\ A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix},
\]

\[
E_{IV} = \begin{bmatrix} 0 & -A_2 & A_1 & -A_3 \\ -A_3 & 0 & 0 & 0 \\ -A_2 & 0 & 0 & 0 \\ -A_1 & 0 & 0 & 0 \end{bmatrix}, \quad F_{IV} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]

in which, \(U\) is the diode voltage drop normalized by the input source. This changes the output matrix of the system because now there is one extra input. The resulting normalized state-space model is defined by the following matrices:

\[
\mathbf{u}(t) = [\mathbf{I}_m, \mathbf{I}_o, \mathbf{V}_D]^T T = \begin{bmatrix} 1, S, Q_t, V_D/V_m \end{bmatrix}^T = \begin{bmatrix} 1, 1/\sqrt{d}, Q_t, U \end{bmatrix}^T,
\]

APPENDIX - NORMALIZED STATE-SPACE MODEL INCLUDING DIODE VOLTAGE DROP

For low voltage levels, the diode \(D\) voltage drop \(V_D\) plays a significant role. In this sense, this section shows the normalized state-space model for the Class-E\(^2\) resonant DC/DC converter. In this case, the state-space models for operating modes II and III, in which the diode \(D\) is off, are going to be enhanced in order to consider the diode voltage drop. The operating modes II and III including the model for the diode voltage drop are depicted in Figure 8(a) and 8(b).

Fig. 8. Operating modes including model for the diode voltage drop. (a) Switch \(S\) on and diode \(D\) on. (b) Switch \(S\) off and diode \(D\) on.

This leads to a third input for the system, which is the diode voltage drop \(V_D\). The input vector becomes:

\[
\mathbf{u}(t) = [\mathbf{I}_m, \mathbf{I}_o, \mathbf{V}_D]^T.
\]

By following the proposed methodology on the manuscript, the input vector is normalized by the input source \(I_m\), which leads to:

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REFERENCES


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